

**Science and Mathematics Education Centre**

**Evaluation of an Innovative Strategy for Teaching Systems of  
Linear Equations in Terms of Classroom Environment, Attitudes  
and Conceptual Development**

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**This thesis is presented for the Degree of  
Doctor of Philosophy  
of  
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## **Declaration**

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university. To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgement has been made.

Signature:

Date:

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## Abstract

This study, which was conducted among middle-school students in California, focused on the effectiveness of using innovative strategies for enhancing the classroom environment, students' attitudes, and conceptual development. Six hundred and sixty-one (661) students from 22 classrooms in four inner city schools completed the modified actual forms of the *Constructivist Learning Environment Survey* (CLES), the *What Is Happening In this Class?* (WIHIC) questionnaire, and the *Test Of Mathematics Related Attitudes* (TOMRA). The data were analyzed for the CLES, WIHIC, and TOMRA to check their factor structure, reliability, discriminant validity, and the ability to distinguish between different classes and groups. In terms of the validity of the CLES, WIHIC, and TOMRA when used with middle-school students in California, the factor analysis results attest to the sound factor structure of each questionnaire. The results for each CLES, WIHIC, and TOMRA scale for the alpha reliability and discriminant validity for two units of analysis (individual and class mean) compare favorably with the results for other well-established classroom environment instruments.

A one-way analysis of variance (ANOVA) was also calculated for each scale of the CLES and WIHIC to investigate its ability to differentiate between the perceptions of students in different classrooms. The ANOVA results suggest that students perceived the learning environments of different mathematics classrooms differently on CLES and WIHIC scales. In general, the results provided evidence of the validity of these instruments in describing psychosocial factors in the learning environments of middle-school mathematics classrooms in California.

The effectiveness of the innovative strategy was evaluated in terms of classroom environment and attitudes, as well as achievement, among a subgroup of 101 students. Effect sizes and *t*-tests for paired sample were used to determine changes in classroom environment perceptions, attitudes, and achievement for experimental and control groups. Pretest-posttest differences were statistically significant ( $p < 0.05$ ) for: the CLES scale of Shared Control for the experimental group, the TOMRA scale of Normality of

Mathematicians for both the control and the experimental groups, the TOMRA scale of Enjoyment of Mathematics for the experimental group, and the achievement measure for both groups. Also ANCOVA was calculated to determine if differential pretest-posttest changes were experienced by the experimental and control groups in classroom environment perceptions, attitudes, and achievement. The results suggest that there were a statistically significant differential changes for Task Orientation, Normality of Mathematicians, Enjoyment of Mathematics, and achievement between the experimental and control groups. In each case, the experimental group experienced larger pretest-posttest changes than the control group.

Overall, a comparison of the pretest-posttest changes for an experimental group, which experienced the innovative strategy, with those for a control group, supported the efficacy of the innovative teaching methods in terms of learning environment perceptions, attitudes to mathematics, and mathematics concept development.

The results of simple correlation and multiple correlation analyses of outcome-environment associations for two units of analysis clearly indicated that there is an association between the learning environment and students' attitudes and mathematics achievement for this group of middle-school mathematics students. In particular, there is a positive and statistically significant correlation between: Normality of Mathematicians and Student Negotiation, Involvement, and Task Orientation with the individual as the unit of analysis; Enjoyment of Mathematics and all three CLES and three WIHIC scales with the student as a unit of analysis, and for the four scales of Personal Relevance, Shared Control, Involvement, and Task Orientation with the class mean as the unit of analysis. The multiple correlations between the group of three CLES and three WIHIC scales and each of the two TOMRA scales are statistically significant for the individual as a unit of analysis. Overall, the study revealed positive and statistically significant associations between the classroom learning environment and students' attitudes to mathematics.

A two-way MANOVA with repeated measures on one factor was utilized to investigate gender differences in terms of students' perceptions of classroom environment and attitudes to mathematics, as well as mathematics achievement. A statistically significant but small difference was found between the genders for Student Negotiation and Task Orientation. Female students perceived their mathematics classrooms somewhat more positively than did the male students. There was no statistically significant difference between the genders on achievement and students' attitudes to mathematics.

Qualitative information, gathered through audiotaped interviews, students' journal, and analysis of students' work, was used to clarify students' opinions about the new approach, classroom environment perceptions, attitudes, and conceptual development. These qualitative information-gathering tools were utilized to obtain a more in-depth understanding of the learning environments (Tobin, Kahle, & Fraser, 1990) and the results of my study (Punch, 1998), as well as insights into students' perceptions (Spinner & Fraser, 2005). The responses from the students' interviews and students' reflective journals from the group that experienced the innovative methods generally suggested that introducing Cramer's rule as a method for solving systems of linear equations in the middle school can be beneficial and therefore might be considered for inclusion in the middle-school Algebra 1 curriculum more widely in California. Using only quantitative data would not have provided the richness that was derived from using mixed methods (Johnson & Onwuegbuzie, 2004). Therefore, qualitative data obtained from students who experienced the innovative method generally supported the quantitative findings concerning the effectiveness of this method for teaching and learning systems of linear equations.

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## **Chapter 1**

### **BACKGROUND TO THE STUDY**

#### **1.1 OVERVIEW**

Mastery of techniques for solving systems of linear equations can empower students to handle many types of real-life problem situations (Larson, Kanold, & Stiff, 1998, p. 344). These real-life situations include, but are not limited to, finance, healthcare, energy, and chemistry. Many scientific and engineering applications which occur in daily life can be ultimately modeled in terms of systems of linear algebraic equations (Mittal & Al-Kurdi, 2001). Also, many real-life situations, such as multiple investment, cost comparison, and comparing two changing quantities and the rate at which they are changing, can be modeled with systems of linear equations (Larson et al., 1998). Therefore, it is very important to learn this topic not only for the above reasons, but also for getting students ready for college mathematics and the life application of the concept in general.

Therefore, the present study evaluated the effectiveness of teaching and learning systems of linear equations (SLE) using innovative methods which included a numerical method (Cramer's rule). This study was conducted among middle-school students in a low socio-economic community in Southern California and focused on the effectiveness of using innovative strategies for enhancing the classroom environment, students' attitudes, and conceptual development.

An overview of this chapter is presented below:

- Introduction (Section 1.2)
- Background to the study (Section 1.3)
- Rationale for the present study (Section 1.4)
- Context of the study (Section 1.5)
- Learning environment (Section 1.6)
- Specific research questions (Section 1.7)



- Outline of other chapters (Section 1.8)
- Significance of the study (Section 1.9)
- Limitations (Section 1.10)
- Summary of the chapter (Section 1.11).

## **1.2 INTRODUCTION**

This research involved the evaluation of the effectiveness of teaching and learning systems of linear equations (SLE) using innovative methods which included a numerical method. Experience has shown that many middle school students in algebra have problems with conceptual understanding of systems of linear equations. This study, which was conducted among middle-school students in California, focused on the effectiveness of using innovative strategies for enhancing the classroom environment and students' attitudes and conceptual development. Also, I investigated how classroom environment is related to students' attitudes and understanding. The Constructivist Learning Environment Survey (CLES), the What Is Happening In this Class? (WIHIC) questionnaire, and the Test of Mathematics-Related Attitudes (TOMRA) were utilized to examine classroom environment and attitudes.

Research on the teaching and learning of systems of linear equations (SLE) at the middle-school level is still in its infant stage. Experience has shown that many high school students in Algebra 1 have problems with conceptual understanding of SLE. The way in which instruction is delivered for some challenging mathematics topics, such as SLE and numerical integration, is often traditional. Traditional methods of teaching typically are teacher-centered with the teacher delivering instruction to a whole group while the students listen and take notes. The demands of new century require that all students acquire an understanding of concepts, skills, and a positive attitude towards mathematics in order to be successful (Kennedy & Tipps, 2000).

My study was conducted to evaluate innovative approaches to the teaching and learning of systems of linear equations in terms of classroom environment, students' attitudes, and their conceptual development. Because systems of linear equations have a very broad application in real life (e.g. modeling of physical and non-physical

systems, and obtaining parameters for decision making in business and design in engineering), the time spent in developing and teaching innovative strategies is likely to be worthwhile in the long run.

The topic of systems of linear equations including numerical methods (Cramer's rule) has neither been taught in the middle-school nor considered for inclusion in the middle-school algebra curriculum in California. In an extensive literature review conducted by Bush, Brown, Ronau, Myers, McGatha, Thompson, Moody, and Karp (2005) regarding what mathematics knowledge middle-school students and teachers should know, the authors showed that systems of linear equations have not received much research attention from mathematics teachers. Several benefits can be derived from using this strategy to reinforce conceptualization and concept change. The use of this approach to teach the solution of SLE could be rewarding and empowering to the students.

In this study, the following approaches involving both quantitative and qualitative research methods were used to collect data:

- The effectiveness of the innovative approach was assessed using three survey instruments – the Constructivist Learning Environment Survey (CLES), What Is Happening In this Class? (WIHIC) questionnaire, and Test of Mathematics-Related Attitude (TOMRA) – as well as achievement and concept map tests.
- To triangulate the findings from the quantitative method, audiotaped interviews with students and students' journals and reflections were collected and analyzed.
- Students' work samples were analyzed so that common errors and misconceptions that were built as they learned systems of linear equations in two variables could be identified and addressed.

### **1.3 BACKGROUND TO THE STUDY**

According to the *Mathematics Framework for California Public Schools, Algebra 1 Standards* (pp. 158-159):

Perhaps the fundamental difficulty for many students making the transition from arithmetic to algebra is their failure to recognize that the symbol  $x$  stands for a number. The first basic skills that must be learned in Algebra I are those that relate to understanding linear equations. In Algebra I the students are expected to solve only two linear equations in two unknowns, but this is a basic skill.

In particular, Standard 9.0 states:

Students solve a system of two equations in two variables algebraically and are able to interpret the answer graphically.

Informed by this and my experience in teaching middle-school algebra, I have come to observe over the years that most students of Algebra I find supposedly basic skills difficult to understand.

In the light of this, many students develop a disinterest in mathematics, lose focus in mathematics classes, and find excuses not to pursue mathematics-related courses at the university. In addition, in higher institutions of learning, the story is no different and can be frightening. Instructional delivery methods might not be encouraging to average students to succeed in mathematics and science classes. Philipou and Christou (1998) used the *Duton scale* to assess the effects of a preparatory mathematics program in changing prospective teacher's attitudes towards mathematics. Their study revealed an alarmingly high proportion of students who brought very negative attitudes to teacher education. These kinds of education students, who will eventually become teachers, might not help in changing the students' attitudes positively in mathematics.

With the *No Child Left Behind* (NCLB) Act of 2001, all eighth-grade students in the United States are expected to take Algebra 1. The *No Child Left Behind* not only advocates for educational reform, but also it is designed to improve student achievement. The NCLB act is designed to change the culture of America's schools by closing the achievement gap between different groups, offering more flexibility, giving parents more options, and teaching students based on what works. The NCLB act also calls for the use of 'scientifically based research' as the foundation for many educational programs and for classroom instruction. According to the act: "America's schools are not producing the mathematics excellence required for global economic leadership and homeland security in the 21st century." The solution suggested by the

act for mathematics education is to ensure that schools use scientifically-based methods with long-term records of success to teach mathematics and measure student progress. In line with this recommendation, my research sought ways to develop and evaluate methods that will overcome some of these problems. In my study, an attempt was made to evaluate a method of teaching challenging topics in a more structured way, starting with the basic foundation and working up to the details. Therefore, for each topic that was taught, the conceptual foundation was laid before the students were taught how to deal with it. The use of technology in designing and delivering instruction was evaluated and incorporated in the research.

#### **1.4 RATIONALE FOR THE PRESENT STUDY**

A number of factors have been found to influence students' approaches to learning (Dart, Burnett, Boulton-Lewis, Smith, & McCrindle, 1999). Several research findings have established that students' perceptions of their learning environments have a significant influence on their approaches to learning, attitudes, and the level of their achievements (e.g. Fraser, 1989, 1998a, 1998b). Many studies have established close connections between students' perceptions of learning and teaching environments, their personal characteristics and attitudes, their approaches to learning and their learning outcomes. The results from my study are likely to contribute towards understanding why middle-school students in California do not achieve proficiently in Algebra 1. Also, the results of the study will provide information regarding the process of improving achievement scores for 8<sup>th</sup> grade algebra students in California.

Subsequent research has shown that students' prior orientations to study and their prior understandings of the subject matter relate to their perceptions of the teaching and learning context with which they are engaged (Crawford, Gordon, Nicholas, & Prosser, 1998). Their work focused on how students' prior understanding of the nature of mathematics relates to their perceptions of learning contexts, approaches to study and subsequent outcomes. Several researchers have reported that the approaches adopted by students to their learning have been shown to be related to their perceptions of learning environment (Entwistle & Ramsden, 1983; Fraser, 1998; Trigwell & Prosser, 1991) and their concepts of learning (Saljo, 1979; Van

Rossum & Schenk, 1984). This means that students who perceive that the teaching is good, that the goals are clear, and that they have some independence in learning are also likely to be adopting a deep approach to learning, while students who perceive that the workload is too high and that the assessment measures rote learning are likely to be adopting a surface approach (Crawford et al., 1998).

In a similar study, Stipek, Salmon, Givvin, Kazemi, Saxe, and Macgyvers (1998) noted that students who have learning goals are more attentive, select more challenging tasks, persist longer in the face of difficulty, use more effective problem-solving strategies, and learn better, especially at a conceptual level, than do students who have performance goals. Therefore, my study examined how the learning environment supports a rigorous algebra topic based on an innovative teaching and learning approaches and how the students' attitudes are affected as they learn this topic. The study provided information that can be adopted and utilized to improve the learning environment in a way that will enhance the students' achievement and attitudes.

In teaching and learning systems of linear equations, some students make errors based on their prior misconceptions. Previous research provides extensive and detailed accounts of student errors and misconceptions in varied domains (Moschkovich, 1999). In mathematics, student errors and misconceptions have been described in the areas of linear equations (Pirie & Martin, 1997), natural number domain to the integers (Gallardo, 2002), whole number subtraction (Brown & Burton, 1978; Brown & Van Lehn, 1981; Burton, 1982; Carpenter et al., 1982), rational numbers (Post et al., 1985), algebra (Matz, 1982; Clement, 1980, 1982),  $x$ -intercept (Moschkovich, 1999), graphs (Bell & Janvier, 1981), concept of conservation of area (Kordaki, 2003), and functions (Herscovics, 1989). Such research documents specific students' errors and misconceptions, describes how they are at variance with expert ideas, and sometimes suggests instructional strategies for addressing particular errors and misconceptions (Moschkovich, 1999). None of these studies have specifically focused on the students' errors and misconceptions in relation to solving systems of linear equations. Moreover, no study has been done on the errors that students make when they solve systems of linear equations by

Cramer's rule at the middle-school level. Therefore, this research was done to correct these misconceptions in teaching and learning systems of linear equations.

Teaching mathematics concepts to students who do not have a very strong background in mathematics is always challenging (Nooriafshar, 2002). From my previous experiences, students have learning problems of which other teachers might not be aware when it comes to solving a system of linear equations by substitution and combination (elimination) methods. In solving SLE by substitution and combination methods, some students make common mistakes which result from their misunderstanding of why and how each step is performed. Analysis of some of the students' work samples has shown me that some students could not recognize when to subtract or add the system; also they do not know when to multiply by a number in order to add or subtract the system.

In traditional mathematics learning and teaching, the teacher is in control and presents rules of mathematics to the students (Thompson, 1992). The traditional approach to mathematics knowledge is a symbolic reconstructive approach and it is developed inside the interaction between the student and the teacher, usually according to a transmissive teaching strategy. Teachers should realize that understanding basic concepts requires that the students be fluent in the basic computational and procedural skills and that this kind of fluency requires practice of these skills over an extended period of time (Bahrack & Hall, 1991; Cooper & Sweller, 1987; Sweller, Mawer, & Ward, 1983). These shortcomings in instructional delivery are what the teaching strategy in this research sought to improve upon.

## **1.5 CONTEXT OF THE STUDY**

This section provides a brief overview of the geography and a description of the education system in the state of California and the schools where my data were collected. The learning context which informed this study is discussed.

### **1.5.1 Education System in California and Students' Demography**

There are three main school types in California: elementary school (Grades K–6); middle school (Grades 7–8); and high school (Grades 9–12). The total enrollment for

the 2003–2004 school year was about 6.3 million. These three main school types are administered and run by school districts. With a population of over 29 million, and ranking the sixth largest economy in the world, California is endowed with huge resources which require a large trained manpower to run.

The survey data used in this study were gathered from four schools within the same school district. The overall student ethnic distribution in the district is: about 73% Hispanic, 24% African American, and 3% other. The students who participated in the data-collection process were 8<sup>th</sup> grade students from inner city schools. The subsample of students who participated in the experimental and control groups for this study were also 8<sup>th</sup> grade middle-school students from a low socio-economic community. The ethnogeographic distribution of these students is about 46% African American, 51% Hispanic and 3% other. The students' ages range between 13 and 14 years. The classes were heterogeneous in terms of previous mathematics achievement scores.

### **1.5.2 Teaching and Learning Context**

The way in which a topic is introduced can have a major impact on the way in which the students receive and perceive a lesson. The attitudes, interests, and learning methods of different students can also play an important role in whether students are ready to acquire a new body of knowledge. The prior knowledge, concepts, and misconceptions of the students can help to build new knowledge and concepts. Despite several decades of innovation in curriculum content and design, there is still scant evidence to suggest that student understanding in mathematics and science has improved (Mansfield & Happs, 1992). Traditional teaching strategies often do not recognize students' conceptions (Mansfield & Happs, 1996). Evans, Midkiff, Morgan, Krausse, Notaros, Rancour, and Wage (2001) claim that these misconceptions, sometimes referred to as 'alternative views' or 'student views' of basic concepts (because they make sense to the student), block the establishment of connections between basic concepts. These connections are necessary for understanding the macro-conceptions developed in further work. This is one of the contexts which informed this study.

Often the materials presented to the students are one-sided because of the inability of the teacher to give the learner a thorough overview of the topic taught to help them to build conceptual ideas, understanding and, finally, mastery. This information is not presented in a coherent structure that will help the students to tie all ideas together. The shortcoming in the strategy of lesson delivery often leads the students to develop apathy towards what is supposed to be an interesting concept and topic. When the teacher presents instruction in a more structured approach to build personal understanding of the topic taught, relates new material to prior knowledge, and uses a variety of instructional strategies (such as hands-on, visual, kinesthetic, audiovisual, and technology including computer-based approaches), it is anticipated that students stand a better chance of improving their conceptualization of systems of linear equations and a numerical method of solving SLE. According to the *Mathematics Framework for California Public Schools* (2000, p. 198)

Algebra I is a gateway course. Without a strong background in the fundamentals of algebra, students will not succeed in more advanced mathematics courses such as calculus.

In this study, different innovative approaches for teaching and learning the solution of systems of linear equations, including a numerical method (Cramer's rule), were evaluated in terms of classroom environment, attitudes, achievement, and concept development. The study compared the effectiveness of an innovative and systematic strategy with traditional methods of teaching systems of linear equations. This involved comparison of a control group and an experimental group.

## **1.6 LEARNING ENVIRONMENTS**

My study drew on and contributed to the field of learning environments. This field involves conceptualizing, assessing, and investigating what happens to students during their schooling (Fraser & Fisher, 1994).

The use of classroom environment inventories to assess learning environments evolved from an evaluation of Harvard Project Physics which required the development of a questionnaire to assess learning environments in physics classrooms. Walberg developed the widely-used Learning Environment Inventory



(LEI) as part of the research and evaluation activities of Harvard Project Physics (Walberg & Anderson, 1968). The Learning Environment Inventory asks students for their perceptions of the whole-class environment (Anderson & Walberg, 1974). Around the same time, Trickett and Moos (1973) had been developing a series of environment measures that included the Classroom Environment Scale (CES), which also asks students for their perceptions of the learning environment. Moos developed the first of his social climate scales, including those for use in psychiatric hospitals and correctional institutes, which ultimately resulted in the development of the Classroom Environment Scale (CES) (Moos, 1979; Moos & Trickett, 1987).

Research on classroom learning environments has evolved since then with researchers developing numerous questionnaires designed to measure perceptions of a range of dimensions pertinent to the learning environment (Fraser, 1998b). The use of students' perceptions of classroom environments as predictor variables has established consistent relationships between the nature of the classroom environment and the student cognitive and affective outcomes (Taylor, Fraser & Fisher, 1997).

The present study used questionnaire data to investigate associations between the learning environment, students' attitudes, and concept development. Two classroom learning environment instruments (the Constructivist Learning Environment Survey, CLES, and the What Is Happening In this Class?, WIHIC, questionnaire) and an attitude questionnaire – the Test of Mathematics Related Attitudes (TOMRA) – were used in the present study. Several recent studies have used these questionnaires in evaluating innovative educational programs (e.g. Chen, Chang & Chang, 2002; Fraser, 1979; Harwell, Gunter, Montgomery, Shelton and West; 2001; Martin-Dunlop, 2003; Spinner & Fraser, 2005), and in investigating the effects of environment on students' learning (McRobbie & Fraser, 1993). Past studies using these questionnaires are of particular relevance in that CLES, WIHIC, and TOMRA are used in the present study (see Chapter 3).

The decision to use CLES and WIHIC in my study was informed by the fact that these two survey instruments have been widely used and exhibited sound validity, reliability, and factor structure. Johnson and McClure (2004) used the CLES to provide insights into the classroom learning environments of beginning science

teachers; Nix, Ledbetter, and Fraser (2005) used the CLES to inform design, guide delivery, and enable multi-level program evaluation; Sebela, Fraser and Aldridge (2004) used teacher action research to promote constructivist classroom environment. In addition, other past workers who used the CLES in their studies include Dorman (2001), Dryden and Fraser (1996), Kim et al. (1999), and Roth and Roychoudhury (1994), just to mention a few. Studies involving the use of CLES internationally include Taiwan (Aldridge, Taylor, Chen & Fraser, 2000), the USA (Fraser & Dryden, 1996, 1998), Nigeria (Idiris & Fraser 1997), and Korea (Kim, Fisher & Fraser, 1999; Lee & Fraser, 2002).

The WIHIC survey also has been used by several researchers in recent studies. Dorman (2003) studied the structural attributes of the WIHIC. Using a large sample of 3,980 students from Australia, Canada, and British high schools, his study demonstrated that the WIHIC can provide a valid measure of classroom environment across several countries. The original version of WIHIC has been validated in Australia and Taiwan (Aldridge, Fraser & Huang, 1999), Taiwan (Chen, Chang & Chang, 2002), Singapore (Fraser & Chionh, 2000), Korea (Kim, Fisher & Fraser, 2000), USA (Rickards, Bull & Fisher, 2001; Rickards, den Brok, Bull, & Fisher, 2003), Indonesia (Margianti, Fraser & Aldridge, 2002), Canada (Zandvliet & Fraser, 2004, 2005), Australia, UK, and Canada (Dorman, 2003). The findings of these studies replicated those of the past research, which reported associations between the learning environment and the students' outcomes. My study used the CLES in addition to WIHIC to evaluate an innovative strategy for teaching and learning systems of linear equations in terms of learning environment and conceptual development.

## **1.7 SPECIFIC RESEARCH QUESTIONS**

- (1) Are questionnaires for assessing (a) classroom environments and (b) attitudes to mathematics valid when used with middle school students in California?
- (2) Is an innovative teaching approach – involving the use of information technology, numerical methods (Cramer's rule), and constructivist methods for the topic of systems of linear equations – effective in terms of promoting:

- (a) a positive classroom environment
  - (b) student attitudes to mathematics
  - (c) student achievement and ability to identify and apply concepts?
- (3) Are there associations between classroom environment and student attitudes to mathematics?
- (4) Are there gender differences in perceptions of classroom environments, attitudes to mathematics, and mathematics achievement?

## **1.8 OUTLINE OF OTHER CHAPTERS**

Chapter 1 discusses the background context of the study, including a brief introduction of learning environments, the specific research questions, and discussion of the significance and limitations of the research. Also, the rationale for my evaluation and teaching of the topic of systems of linear equations is discussed.

Chapter 2 is a review of literature about various learning environment inventories, scales, and dimensions. This includes a review of classroom learning environment from a theoretical and historical perspective, as well as the specific learning environments questionnaires utilized in this study. The literature on qualitative and quantitative research methods utilized in this study is also reviewed. A brief overview of a range of past studies that used learning environment as criteria in evaluating educational program and in outcome-environment associations were reviewed in detail in this chapter. In addition, a brief history of systems of linear equations, including solution methods, is reviewed. In particular, the Cramer's rule method and literature about these topics are reviewed.

Chapter 3 focuses on research methodology. In this chapter, the data-collection processes, data sources, and research paradigm are presented. Also, the samples for the control and experimental groups are described. The quantitative and qualitative data sources, methods of data coding and analysis, case studies, and interviews are discussed. In this chapter, an overview on the use of qualitative and quantitative methods within the same study in research on learning environments is presented.

Chapter 4 reports findings from quantitative data in terms of the factor structure, validity, and reliability of the instruments, a comparison of the control and experimental groups in terms of achievement, attitudes, and classroom environment, and associations between student outcomes and classroom environment. Gender difference in terms of learning environment perceptions and attitudes to mathematics are reported.

Chapter 5 presents findings from qualitative data, including case studies, interviews, students' journal writing, and concept maps, including error analyses and comparisons between the experimental and control groups. This chapter discusses the audiotaped interview obtained from both the students who experienced the innovative strategy and the control group. Also, the information gathered from students' learning journals is analyzed and presented in this chapter. The analysis of students' work samples is reported and the findings involving qualitative data are presented.

Further discussion of my research can be found in Chapter 6, which also concludes a summary of the present study. Implications of the findings from this research for improving classroom environment and students' attitudes to mathematics are presented, together with limitations and suggestions for future research. This chapter also addresses the research findings involving the use of Cramer's rule to teach systems of linear equations and the reasons for considering this method in middle-schools curriculum.

## **1.9 SIGNIFICANCE OF THE RESEARCH**

This study can be viewed as educationally important in that it evaluated a stimulating classroom environment that makes use of variety and interesting strategies for learning, such as computer-assisted approaches. The teaching and learning of systems of linear equations involving a numerical method has never been undertaken in the middle-school in California. Also these topics have not been considered for inclusion in the middle school algebra curriculum.

In order to help to prepare the students conceptually to be adaptable to college learning and challenges, an attempt was made to incorporate innovative mathematics teaching and learning into the middle-school curriculum early enough to accommodate the students' learning needs. In their study, Crawford et al. (1998) were concerned with how mathematics students' prior conceptions of the nature of the subject matter that they were about to study interacted with other aspects of their prior experiences and understandings, and how these conceptions related to their approaches to and perceptions of their subsequent experiences and understanding.

My research is important also in that there is only a limited number of learning environment studies internationally that has focused specifically on mathematics classes (e.g. Spinner & Fraser, 2005; Majeed et al., 2002). None of this handful of learning environment studies has focused primarily on the teaching and learning of systems of linear equations and associations with the students' attitudes and conceptual development.

This was one of the initial studies in California using the What Is Happening In this Class? (WIHIC) questionnaire with eighth-grade mathematics classes. This study is expected to contribute to the field of learning environment by evaluating whether learning environment questionnaires are valid when used with middle-school students in California. Finally, this study is distinct in that it did not rely only on achievement. Student attitudes to mathematics and their classroom environment perceptions also were main foci of this research (e.g. Margianti et al., 2002; Spinner & Fraser, 2005). Also, it evaluated the effectiveness of an innovative teaching approach and early inclusion of challenging topics in the curriculum.

## **1.10 LIMITATIONS**

However, as with all research, my study had some limitations as well as strengths. For example, the teacher being the researcher in this study might have led to bias and to making errors of judgment in assessing the students because the researcher might not have been an impartial observer. It was not always possible to have other teachers of mathematics observe the experimental group classes during the study. If this were possible, their feedback and critique would have been valuable in

enhancing the quality of this study. Seeing what the researcher wants to see could have obscured the sound judgment of the researcher, especially in qualitative data gathering. Spinner and Fraser (2005) encountered and reported this problem in their study.

The representativeness of the sample could be another limiting factor in that, when compared to the general eighth-grade population in California, my sample could be considered neither a sizeable fraction of the population nor representative of the full range of schools and students. This limits the generalizability of findings. Student cultural background is another variable that was not considered in this research. Some students are African Americans, while others are limited English proficient who might not be in tune with the constructivist approach of teaching and learning – on which the innovative method is based. For instance, the sociocultural background of most of the students in my school community might make it difficult for them to adapt to the new teaching and learning approach. The findings from this study might not be generalizable to other cultural backgrounds.

The statistical power could be limited in some data analyses in this study due to the sample size of 661 students. In particular, out of these 661 students, a subsample of only 101 students comprised the control and the experimental groups, thus making the lack of statistical power especially relevant for analyses involving this subsample. In addition to the limitations associated with the quantitative data, the extent of the qualitative component of this study was limited in that relatively few students ( $N=12$ ) were interviewed. Conducting extensive and comprehensive qualitative data collection would have been preferable. Nonetheless, an attempt was made to obtain students' journal writing to reduce this shortcoming.

## **1.11 SUMMARY OF THE CHAPTER**

This chapter introduced the research including the rationale for the present study. The research questions were delineated and the significance of the study was established. The limitations of the present study were also presented, together with how these limitations might have affected the validity and generalizability of the

results obtained from the present study. In summary, Chapter 1 focused on introducing the background to the study according to the following headings:

- Introduction (Section 1.2)
- Background of the research (Section 1.3)
- Rationale for the present study (Section 1.4)
- Context of the study (Section 1.5)
- Learning environment (Section 1.6)
- Specific research questions (Section 1.7)
- Outline of other chapters (Section 1.8).

While this chapter aimed to place the work done in the present study in perspective, a more comprehensive literature review is presented in the next chapter.

## **Chapter 2**

### **LITERATURE REVIEW**

#### **2.1 OVERVIEW**

My study involved investigation of an innovative strategy for teaching and learning of systems of linear equations in terms of classroom learning environment, students' attitudes to mathematics and conceptual development in California middle schools. Therefore, this chapter reviews classroom learning environment questionnaires from a historical perspective and looks at various classroom learning environment inventories, scales, and dimensions. In this chapter, also, a historical background of research involving classroom learning environments is presented. The rationale for my research involving an evaluation of an innovative strategy for learning and teaching the topic of systems of linear equations in terms of classroom environment, attitudes, and conceptual development is discussed. Also, the questionnaires that were used in this study are reviewed and a historical background of research utilizing these questionnaires is presented. This includes both cross-national and national studies involving the Constructivist Learning Environment Survey (CLES), the What Is Happening In this Class? (WIHIC) questionnaire, and the Test Of Mathematics Related Attitudes (TOMRA) questionnaire. As my study involved the use of learning environment in evaluating an innovative strategy, past studies that used learning environment as criteria in evaluating educational program and outcome-environment associations are reviewed. Because a concept map was used as one of the tools to monitor the students' conceptual development in this study, a brief description of the history of concept maps and their usefulness are discussed. Literature about the application of computers in the teaching and learning of systems of linear equations is reviewed.

My study drew on an existing educational research paradigm to evaluate the teaching and learning of systems of linear equations in terms of classroom learning environments, attitudes, and conceptual development. This research will contribute to the field of learning environments by evaluating an innovative teaching and



learning approach involving the use of information technology, a numerical method (Cramer's rule), and constructivist learning strategies.

The following is an overview of this chapter:

- Theoretical and historical background of learning environment (Section 2.2)
- Qualitative and quantitative research methods (Section 2.3)
- Learning environment questionnaires (Section 2.4)
- Past research on learning environments (Section 2.5)
- Assessment of attitudes to mathematics (Section 2.6)
- Concept maps (Section 2.7)
- Computer applications for teaching and learning SLE (Section 2.8)
- Teacher-as-researcher (Section 2.9)
- Conclusion (Section 2.10)

## **2.2 THEORETICAL AND HISTORICAL BACKGROUND**

My study drew on and contributed to the field of learning environments (eg. Aldridge, Fraser & Huang, 1999; Fraser, 1994, 1998a, 1998b; Fraser & Tobin, 1991; Maor & Fraser, 1996; Tobin & Fraser, 1998; Tobin, Kahle & Fraser, 1990), including recent studies (eg. Bloese & Fisher, 2003; Johnson & McClure, 2004; Raaflaub & Fraser, 2002; Rickards, den Brok, Bull & Fisher, 2003; Sebela, Fraser & Aldridge, 2004; Spinner & Fraser, 2005; Zandvillet & Fraser, 2004, 2005) involving an evaluation of innovative strategies for teaching and learning. Several research articles have focused on associations between outcomes and environment, evaluation of innovative teaching approaches, teachers' use of student perceptions in guiding improvements in classrooms, differences between students' and teachers' perceptions of the classrooms, determinants of classroom environment, combining quantitative and qualitative methods, links between different educational environments, and cross-national studies (Fraser, 1998a; Fraser, 2002).

The work on educational environments over the previous 35 years builds upon pioneering ideas of Lewin and Murray and those who came after them, such as Pace

and Stern (1958). Lewin's (1936) seminal work on field theory recognized that both the environment and its interaction with personal characteristics of the individual are potent determinants of human behavior. The Lewinian formula,  $B = f(P, E)$  was first enunciated to stress the need for new research strategies in which behavior is considered to be a function of the person and environment. Murray (1938) followed Lewin's approach by proposing a needs-press model which allows the analogous representation of person and environment in common terms.

Earlier work on learning environments built on these momentous theoretical, conceptual, and measurement foundations. Walberg and Moos originated research involving the use of learning environment questionnaires when he developed the widely-used Learning Environment Inventory (LEI) as part of the research and evaluation activities of Harvard Project Physics (Walberg & Anderson, 1968). Simultaneously, Moos began developing the first of his social climates scales, including those for use in psychiatric hospitals and correctional institutions, which ultimately resulted in the development of the Classroom Environment Scale (CES) (Moos, 1979; Moos & Trickett, 1987).

Since Walberg's and Moos' earlier work on learning environment, many contemporary workers have continued to carry out studies in this field. This pioneering work on perceptions of classroom environment developed into major research programs and spawned other research (Fraser, 1998b). Research in the field of learning environments has been catalogued into books (eg. Fraser, 1986; Fraser & Tobin, 1998; Fraser & Walberg, 1991; Fisher & Khine, 2003; Goh & Khine, 2002; Moos, 1979; Walberg, 1979; Wubbels & Levy, 1991), literature reviews (Fraser, 1994, 1998a, 1998b; 2002; MacAuley, 1990; von Saldern, 1992), journal articles (e.g. Aldridge, Fraser & Huang, 1999; Fraser, 1998b; Johnson & McClure, 2004; Yarrow, Millwater & Fraser, 1997), editorials (eg. Fraser, 2001), and conference papers (e.g. Dorman, 2001, 2003; Raaflaub & Fraser, 2002; Rickards, Bull & Fisher, 2001) just to mention but a few that are related to the present study.

Each individual scale in any environment-measuring instrument can be categorized according to Moos' (1974) scheme for classifying human environments. Moos' basic types of dimensions are *Relationship Dimensions* (which identify the nature and

intensity of personal relationships within the environment and assess the extent to which people are involved in the environment and support and help each other), *Personal Development Dimensions* (which assess basic directions along which personal growth and self-enhancement tend to occur) and *System Maintenance and System Change Dimensions* (which involves the extent to which the environment is orderly, clear in expectations, maintains control and is responsive to change. Fraser (1998b) illustrates how the individual dimensions in numerous widely-used classroom learning environment questionnaires can be classified according to Moos' scheme.

Previous research has used learning environment questionnaires in curriculum evaluation (Chen, Chang & Chang, 2002; Fraser, 1979; Mink & Fraser, 2005; Spinner & Fraser, 2005), investigating the effects of environment on students learning (McRobbie & Fraser, 1993), studies of differences between students' and the teacher's perceptions of the same classroom (Fisher & Fraser 1983), research on the transition from primary to secondary school (Ferguson & Fraser 1996, 1999), teachers' practical attempts to improve classroom learning environments (Thorp, Burden & Fraser 1994), research on special education classrooms in England (Adams, 2000), comparisons of actual and preferred environments (Fraser, 1998b; Spinner & Fraser, 2005), and school psychology (Burden & Fraser, 1993). Other contemporary research involving classroom environments include studying science classroom environments in Korea (Kim, Fisher & Fraser, 1999), evaluation of an innovative university science course (Martin-Dunlop, 2003, 2005), and identifying differences in the perceptions of the learning environment between city and country students (Waldrup & Fisher, 2000) and according to school racial diversity and socioeconomic status (Rickards et al., 2001).

### **2.3 QUANTITATIVE AND QUALITATIVE RESEARCH METHODS**

Quantitative and qualitative approaches have been used extensively in recent classroom environment studies, including the evaluation of innovative educational programs (Spinner & Fraser, 2005), investigation of the learning environment in Canadian mathematics and science classrooms (Raaflaub & Fraser, 2002), evaluation of teachers' use of students' environment perceptions in guiding changes in

classrooms (Blose & Fisher, 2003; Sinclair & Fraser, 2003), studying parents' perceptions of classroom environments (Robinson & Fraser, 2003), use of teacher action research to improve classroom environment (Sebela, Fraser, & Aldridge, 2004), and establishing the validity and reliability of a shortened, revised version of CLES (Johnson & McClure, 2004).

The combination of quantitative and qualitative methods can be termed mixed method. Johnson and Onwuegbuzie (2004, p. 17) defined mixed-methods research as mixing or combining quantitative and qualitative research techniques, methods, approaches, concepts or language into a single study. They claim that researchers can put together insights and procedures from both approaches to produce a superior product. Multiple methods in research are useful in achieving greater understanding (Denzin & Lincoln, 1994; Tobin & Fraser, 1998). In learning environment research, considerable progress has been made in realizing the benefits of combining quantitative and qualitative information methods (Dorman, Fraser & McRobbie, 1994; Fraser & Tobin, 1991; Maor & Fraser, 1996; McRobbie, Fisher & Wong, 1998; Tobin & Fraser, 1998; Tobin, Kahle & Fraser, 1990). Recently, many researchers have combined qualitative and quantitative methods in classroom environment research (e.g. Aldridge, Fraser & Huang, 1999; Blose & Fisher, 2003; Harwell, Gunter, Montgomery, Shelton & West, 2001; Spinner & Fraser, 2005). For example, student perceptions as measured by the CLES were combined with teacher logs, teacher interviews, and field notes from team discussions in the classroom (Harwell et al., 2001).

Fraser and Tobin (1991) illustrate the merits of combining qualitative and quantitative methods in learning environment research by drawing on three case studies of successful attempts at using questionnaire surveys and ethnographic methods together within the same investigations in science education. The three case studies focused on (1) higher-level cognitive learning, (2) the nature and role of 'target' students who dominate classroom discourse, and (3) exemplary teachers. Also, another advantage of mixed-methods research (Johnson & Ownuegbuzie, 2004) – as they chose to call the combining of quantitative and qualitative methods – is that multiple approaches are used in answering research questions, rather than restricting or constraining researchers' choices. It is an expansive and creative form

of research, not a limiting form of research according to Johnson and Onwugbuzie. They added that mixed-method research is inclusive, pluralistic, and complementary.

The use of qualitative methods in learning environment research (Tobin, Kahle, & Fraser, 1990) also has provided a more in-depth understanding of learning environments. Tobin and Fraser (1998) used multiple theoretical perspectives to frame the research and its methods, and to illustrate the desirability of combining quantitative and qualitative data to maximize the potential of research on learning environments. Blose and Fisher's (2003) study involved teachers integrating theory with practical work to produce positive student outcomes and productive classrooms by assessing, describing, and changing their mathematics classroom environments. They utilized a triangulation of quantitative and qualitative data sources by noting that a more complex view of elementary mathematics classroom learning environments could be obtained.

Aldridge, Fraser and Huang (1999) drew on multiple research methods that were combined to help in examining and comparing science classroom learning environments in Taiwan and Australia from different perspectives. They used triangulation to secure an in-depth understanding of the learning environment and to provide richness to the whole. Using a large sample provided an overview of the learning environment in each country. However, they found that the data posed more questions than it answered. A sense of the problem was developed during observations that reshaped the inquiry towards an examination of socio-cultural influences that might affect what was considered to be a desirable learning environment in each country (Aldridge, Fraser & Huang, 1999, p. 50). The data which they collected using the questionnaires were then used as a springboard for further data collection involving qualitative methods. My study combined quantitative and qualitative methods (mixed approach) in an attempt to produce more insightful research.

### **2.3.1 Quantitative Methods**

The use of quantitative method in learning environment research has been widely reported. Past research using quantitative method as a research approach include

Margianti, Fraser and Aldridge (2002), in assessing the perceptions of Indonesian university students; Newby and Fisher (1997), in assessing the learning environment of a computer laboratory; Ferguson and Fraser (1999), in investigating changes in learning environment during the transition from primary to secondary school; Taylor, Fraser and Fisher (1997), in monitoring constructivist classroom learning environments; Dorman (2001), in investigating associations between classroom environment and academic efficacy; and Majeed, Fraser and Aldridge's (2002) in study of the associations of learning environment with student satisfaction among mathematics students. These studies employed statistical analysis in analyzing survey and questionnaire data which enabled the researchers to discuss and explain the classroom environments under study. Therefore, my study incorporated this method as part of the approaches used in gathering data.

### **2.3.2 Qualitative Methods**

Numerous past learning environment researchers have used qualitative methods as part of a mixed approach in providing rich insights into the overall perceptions of the students in the classroom (e.g. Fraser & Tobin, 1991; Nix et al., 2005; Raflaub & Fraser, 2002). Consequently, there are many methods for investigating students' learning. Duit, Treagust, and Mansfield (1996) organized methods for investigating students' learning and understanding in terms of whether they involve naturalistic settings, interviews, conceptual relationships, diagnostic test items, or computerized diagnosis. These methods have been used in classroom learning environment studies in order to qualitatively understand and explain students' perceptions of their learning environment, outcome-associations, and attitudes. Burns (1997) employed research methods that drew on ethnographic techniques such as participant observation, interviews, survey, and the collection of video and auditory records in the investigation of students' and teachers' use of technology in specific classroom environments. In my study, ethnographic techniques, such as using concept maps and naturalistic settings (collection of video and auditory records), were used in conjunction with survey instruments to investigate holistically students' understanding of the solution of systems of linear equations in two variables.

Whereas qualitative methods were used in my study to investigate students' understanding of the systems of linear equations, they also provided insights into students' perceptions of their psychosocial learning environment, attitudes to mathematics, and conceptual development. In this study, qualitative methods involving audiotaped interviews, journal and reflection writing, and analysis of students' work samples were used in obtaining information which complemented the quantitative data in order to triangulate the methods. When a study using quantitative methods has been completed, its main findings can be contextualized with thick descriptions consisting of observations and verbal accounts from participants (Tobin & Fraser, 1998).

In naturalistic settings, teaching and learning events are often audiotaped or videotaped and verbatim transcripts are made of the lesson or activity (Duit et al., 1996). Their view is that field notes of the lesson or activity taken by the researcher can be written up later to provide an intensive account of the observations. Johnson and McClure (2004), Sebela et al. (2004), and Harwell et al. (2003) used observations in naturalistic settings and in classrooms to investigate students' ideas in mathematics and science classrooms and to develop new ways of teaching within a constructivist framework. Solomon (1985) noted that detailed descriptions of lessons not only provide useful information about students' ideas of the concepts taught, but they also make teachers more sensitive to their students' way of thinking in general.

## **2.4 LEARNING ENVIRONMENT QUESTIONNAIRES**

There are different classroom and learning environment inventories and questionnaires that now abound for the assessment of learning environments (e.g. Fraser, 1998b). Many of these inventories resulted from the adaptation of the existing questionnaires. Newby and Fisher (1997), for example, described the adaptation of an instrument in their study for use in computer laboratory learning environments in higher education.

This section is devoted to reviewing literature on classroom environment questionnaire using the following headings:

- Overview of a variety of questionnaires (Section 2.4.1)
- Constructivist Learning Environment Survey (Section 2.4.2)
- What Is Happening In this Class? (WIHIC) questionnaire (Section 2.4.3).

### 2.4.1 Overview of a Variety of Questionnaires

Some of the historically-important and contemporary learning environment instruments include: Learning Environment Inventory (LEI); Classroom Environment Scale (CES); Individualized Classroom Environment Questionnaire (ICEQ); My Class Inventory (MCI); College and University Classroom Environment Inventory (CUCEI); Questionnaire on Teacher Interaction (QTI); Science Laboratory Environment (SLEI); Constructivist Learning Environment Survey (CLES); and What Is Happening In this Class? (WIHIC) questionnaire. Table 2.1 depicts for each instrument the name of each scale, the level (elementary, secondary, higher) for which each instrument is suited, the number of items contained in each scale, and the classification of each scale according to Moos' (1974) scheme for classifying human environments (Fraser, 1998b).

In this table, Moos' three basic types of dimensions are presented. *Relationship Dimensions* identify the nature and intensity of personal relationships within the environment and assess the extent to which people are involved in the environment and support and help each other, *Personal Development Dimensions* assess basic directions along which personal growth and self-enhancement tend to occur, and *System Maintenance and System Change Dimensions* involve the extent to which the environment is orderly, clear in expectations, maintains control and is responsive to change. Each of the first seven questionnaires listed in Table 2.1 is described in turn briefly below.



Table 2.1. Overview of Scales Contained in Nine Classroom Environment Instruments (LEI, CES, ICEQ, MCI, CUCEI, QTI, SLEI, CLES and WIHIC)

Instrument	Level	Items per scale	Scales Classified According to Moos's Scheme		
			Relationship dimensions	Personal development dimensions	System maintenance and change dimensions
Learning Environment Inventory (LEI)	Secondary	7	Cohesiveness Friction Favoritism Cliquesness Satisfaction Apathy	Speed Difficulty Competitiveness	Diversity Formality Material Environment Goal Direction Disorganization Democracy
Classroom Environment Scale (CES)	Secondary	10	Involvement Affiliation Teacher Support	Task Orientation Competition	Order and Organization Rule Clarity Teacher Control Innovation
Individualized Classroom Environment Questionnaire (ICEQ)	Secondary	10	Personalization Participation	Independence Investigation	Differentiation
My Class Inventory (MCI)	Elementary	6–9	Cohesiveness Friction Satisfaction	Difficulty Competitiveness	
College and University Classroom Environment Inventory (CUCEI)	Higher Education	7	Personalization Involvement Student Cohesiveness Satisfaction	Task Orientation	Innovation Individualization
Questionnaire on Teacher Interaction (QTI)	Secondary/ elementary	8–10	Helpful/Friendly Understanding Leadership Student Responsibility and Freedom Dissatisfied Admonishing Uncertain Strict		
Science Laboratory Environment Inventory (SLEI)	Upper Secondary/ Higher Education	7	Student Cohesiveness	Open-Endedness Integration	Rule Clarity Material Environment
Constructivist Learning Environment Survey (CLES)	Secondary	7	Personal Relevance Uncertainty	Critical Voice Shared Control	Student Negotiation
What Is Happening In this Class? (WIHIC)	Secondary	8	Student Cohesiveness Teacher Support Involvement	Investigation Task Orientation Cooperation	Equity

\*Adapted from Fraser (1998b)

#### 2.4.1.1 *Learning Environment Inventory (LEI)*

Walberg is credited with developing the Learning Environment Inventory (LEI) to evaluate a new curriculum called Harvard Project Physics in the late 1960s (Walberg & Anderson, 1968). The final version of the LEI contains 105 statements (seven per scale) descriptive of typical school classes. The LEI was validated statistically based on 1,048 students, except for discriminant validity data which were based on 149 class means (Fraser, Anderson & Walberg, 1982).

#### 2.4.1.2 *Classroom Environment Scale (CES)*

The environment in hospitals, prisons, universities, and work sites was the stepping stone for Moos to develop the Classroom Environment Scale (CES) (Moos, 1979; Moos & Trickett, 1987). The development of this questionnaire was part of Moos' several social climate surveys for use in his work in various human environments including psychiatric hospitals. The final published version contains nine scales with 10 items of True-False response format in each scale. Fisher and Fraser (1983b) validated the CES with 1,083 students.

#### 2.4.1.3 *Individualized Classroom Environment Questionnaire (ICEQ)*

Rentoul and Fraser (1979) developed the Individualized Classroom Environment Questionnaire (ICEQ) to assess dimensions that distinguish individualized classrooms from conventional ones. The long form of the final versions of the ICEQ, the (Fraser, 1990) contains 50 items measuring five dimensions – Personalization, Participation, Independence, Investigation, and Differentiation. The shortened version contains 25 items consisting five scales with each containing five items. The ICEQ has been validated with a sample of 1,849 students (Fraser, 1990).

#### 2.4.1.4 *My Class Inventory (MCI)*

The My Class Inventory (MCI) is a simplified form of LEI for use among children aged 8–12 years (Fisher & Fraser, 1981; Fraser et al., 1982; Fraser & O'Brien, 1985). The MCI was developed originally for use at the primary school level, but it also has been found to be useful with students in the junior high school, especially those with limited reading skills. The MCI is a one-page questionnaire that measures five dimensions of Satisfaction, Friction, Competitiveness, Difficulty, and Cohesiveness and contains 25 questions. The MCI has been validated with 2,305 students in

Australia (Fisher & Fraser, 1981) and in Brunei Darussalam with a sample of 1,565 students in 81 mathematics classes from 15 secondary schools (Majeed, Fraser & Aldridge, 2002).

#### 2.4.1.5 *College and University Classroom Environment Inventory (CUCEI)*

The College and University Classroom Environment Inventory (CUCEI) was designed specifically for the higher education level for use in small classes, such as seminars (Fraser & Treagust, 1986; Fraser, Treagust & Dennis, 1986). The CUCEI assess the seven dimensions of Personalization, Involvement, Cohesiveness, Satisfaction, Task Orientation, Innovation and Individualization. The final form of the CUCEI contains seven seven-item scales. Each item has four responses (Strongly Agree, Agree, Disagree, Strongly Disagree) and the polarity is reversed for approximately half of the items.

#### 2.4.1.6 *Questionnaire on Teacher Interaction (QTI)*

The Questionnaire on Teacher Interaction (QTI) was developed by a team of researchers in The Netherlands and focuses specifically on interpersonal relationships between teachers and their students (Wubbels, Créton & Hoomayers, 1992; Wubbels & Levy, 1993). The QTI can be used to map students' and teachers' perceptions using a model for interpersonal teacher behavior (Wubbels, 1993). One advantage of the QTI is that it can be used to obtain either students' or teachers' perceptions of interpersonal behavior (Fisher, Rickards, & Fraser, 1996). QTI has been validated in the US with a sample of 1,606 students and 66 teachers (Wubbels & Levy, 1991), Australia with a total of 792 students and their 46 teachers (Wubbels, 1993), and The Netherlands with 1,105 students in 66 Grade-nine physics classes (Wubbels, 1993). Also, Fisher, Fraser and Wubbels (1992) validated the QTI with a total of 3,994 high school science and mathematics students. In Quek, Wong, and Fraser's (2005) study of gifted students' attitudes towards chemistry in laboratory classrooms in Singapore, the QTI was validated with a sample of 497 students from three independent schools. Recently, in Thailand, Santiboon (2005) validated the QTI with a large sample of 4,576 students in 245 physics school classes, and Kijkosol and Fisher (2005) validated a Thai version of QTI with 1,194 students in Grade 10 biology classes.

#### 2.4.1.7 *Science Laboratory Environment Inventory (SLEI)*

Science Laboratory Environment Inventory (SLEI) was developed specifically to assess students' practical work experience in science laboratories because of the critical importance and uniqueness of laboratory settings in science education (Fraser, Giddings & McRobbie, 1995; Fraser & McRobbie, 1995). The SLEI has five scales each with seven items. The SLEI was initially developed, field tested, and validated with a sample of 5,447 students in 269 classes in a large cross-national study that involved six countries – USA, Canada, Australia, England, Israel, and Nigeria (Fraser et al., 1995; Fraser & Griffiths, 1992; Fraser & Wilkinson, 1993). Recently, Martin-Dunlop (2003) used selected scales from the SLEI in a study of understanding of the nature of science among a sample of 230 university students undertaking a science course for prospective elementary school teachers.

My study used two important learning environment questionnaires in evaluating an innovative strategy for teaching and learning systems of linear equation: the Constructivist Learning Environment Survey (CLES) and the What Is Happening In this Class? (WIHIC). Therefore, literature relevant to these two instruments is reviewed in detail in Section (2.4.2) and Section (2.4.3).

### 2.4.2 **Constructivist Learning Environment Survey (CLES)**

#### 2.4.2.1 *Development and Description of CLES*

The notion of a constructivist classroom learning environment originates from the strong instructional imperatives of some of the greatest thinkers in education: John Dewey, Jean Piaget, Lev Vygotsky, Howard Gardner and others. Brooks and Brooks (1993, 1999) place the learner in the pivotal role of constructing mental schemes of one's own understanding of the world. Learning from the constructivist viewpoint is "a self-regulatory process of struggling with the conflict between existing personal models of the world and discrepant new insights, constructing new representations and models of reality as a human meaning-making venture with culturally developed tools and symbols, and further negotiating such meaning through cooperative social activity, discourse, and debate" (Fosnot, 1996, p. ix). This conception necessitates teaching approaches which offer students opportunities for concrete, contextually-meaningful experiences in which students search for patterns,

generate their own questions, and construct their own models, concepts, and strategies. Lorschach and Tobin (1992, p. 21) define constructivism as “an epistemology, a theory of knowledge used to explain how we know”.

Meaningful learning is a cognitive process in which individuals make sense of the world in relation to the knowledge which they already have constructed, and this sense-making process, according to the constructivist view, involves active negotiation and consensus building. The Constructivist Learning Environment Survey (CLES; Taylor, Dawson & Fraser, 1995; Taylor, Fraser & Fisher, 1997) was developed to assist researchers and teachers to assess the degree to which a particular classroom’s environment is consistent with a constructivist epistemology, and to assist teachers to reflect on their epistemological assumptions and to reshape their teaching practice. The CLES can be used to assess the level of constructivist teaching and learning practices (Fraser, 1998a, pp. 534-535).

The decision to incorporate the CLES in my study was informed by the fact that this instrument has been widely validated using factor and reliability analyses. Johnson and McClure (2004) used the CLES to provide insights into the classroom learning environments of beginning science teachers. Their study led to the revision and shortening of the CLES based on exploratory factor analysis and internal consistency reliability analysis. Recent studies that have utilized the CLES include: Nix, Fraser, and Ledbetter (2005) who used the CLES to inform the design, guide delivery, and enable multi-level program evaluation; and Sebela et al. (2004) who used teacher action research to promote constructivist classroom environment in South Africa.

Other workers who used the CLES in their studies include Spinner and Fraser (2005), Harwell et al. (2001), Dorman (2001), Dryden and Fraser (1996, 1998), and Roth and Roychoudhury (1994). Also, Kim et al. (1999) studied the assessment of the constructivist science classroom in Korea. The CLES has been used in qualitative studies of the nature of science knowledge and learning of science teachers and their students (Lucas & Roth, 1996; Roth & Bowen, 1995), a study of preservice science teachers’ self-efficacy and science anxiety (Watters & Ginns, 1994), and a study of secondary preservice teachers’ beliefs (Waggett, 2001). Studies involving the use of the CLES internationally include Taiwan (Aldridge, Taylor, Chen & Fraser, 2000),

the USA (Fraser & Dryden, 1996, 1998), and Nigeria (Idiris & Fraser, 1997), and Korea (Kim, Fisher & Fraser, 1999; Lee & Fraser, 2002)

As part of their quantitative information-gathering technique, Spinner and Fraser (2005) used the CLES to evaluate the *Class Banking System* – which is an innovative mathematics program that is designed based on the constructivist view that defines meaningful learning as a cognitive process in which students make sense of mathematical concepts in relation to the mathematical knowledge which they already have constructed. They chose the CLES to assess the level of constructivist teaching and learning practices (Spinner & Fraser, 2005). The CLES, according to Aldridge, Fraser, Taylor and Chen (2000), provides valuable information in its own right. The scales of the CLES were designed to obtain measures of students' perceptions of the frequency of occurrence of five key dimensions of a critical constructivist learning environment: Personal Relevance, Uncertainty of Mathematics, Critical Voice, Shared Control, and Student Negotiation.

Harwell et al. (2001) used the set of five CLES scales to assess students' perceptions of the classroom learning environment by monitoring the alignment of classroom learning activities with a constructivist viewpoint while integrating technology into the curriculum. They explored students' perceptions as measured by the CLES. However, they discovered that there was no significant change in students' perceptions of the classroom learning environment over the duration of the academic year. Dorman (2001) conducted a study of associations between classroom psychosocial environment and academic efficacy involving using scales from CLES and WIHIC. He used three CLES scales in addition to the WIHIC scales to provide a comprehensive assessment of classroom environment. However, the three CLES scales did not explain much unique variance in academic efficacy based on commonality analysis. Taylor, Fraser and Fisher (1997), in another study utilizing CLES, examined the viability of a new version of the CLES for monitoring constructivist transformations to the epistemology of school science and mathematics classrooms as part of their design process.

Because the CLES is not subject specific, revised versions of the CLES for science (CLES–Science) and for mathematics (CLES–Mathematics) (Taylor et al., 1994)

were used specifically to ascertain students' perceptions of science and mathematics classroom learning environments. The perceived version of the CLES measures the extent to which students perceive their learning environment to be consistent with a constructivist epistemology and is designed to assist teachers in reflection on their beliefs and reshaping their teaching practices (Fraser, 1994; Taylor & Fraser, 1991; Taylor, Fraser & Fisher, 1997). The CLES is not subject-specific and includes 28 items with seven items in each of four dimensions. Students respond using a five-point frequency response scale with the responses of Very Often, Often, Sometimes, Seldom, and Never (Harwell et al., 2001).

The revised version of the CLES–Science and the CLES–Mathematics (Taylor et al., 1994) is based on the theoretical framework of critical constructivism and measures students' perceptions of five dimensions of the learning environment. Each instrument has a total of 30 items with six items in each of the five dimensions. Students respond with one of the following frequency choices: Almost Always, Often, Sometimes, Seldom, and Almost Never. Responses correspond to a numeric score where a high score is indicative of a more constructivist environment. The following is a brief description of each subscale and a sample item from each:

- The Personal Relevance (PR) dimension measures students' perceptions of the extent to which either science or mathematics knowledge is connected to students' actual out-of-school experiences (e.g. "I learn how science can be part of my out-of-school life").
- The Uncertainty (U) scale is concerned with the extent to which students experience mathematical or scientific knowledge as coming from ever-changing human experiences and values which are culturally and socially determined (e.g. "I learn that science cannot provide perfect answers to problems").
- The Critical Voice (CV) dimension measures the extent to which students perceive whether it is beneficial and legitimate to question the pedagogical plans and methods which teachers use and to express concerns about any perceived impediments to their learning (e.g. "It's OK to ask the teacher 'Why do we have to learn this?'"').

- Shared Control (SC) scale assesses the extent to which students perceive that they are being invited to share control with the teacher of the total learning environment, including the design and management of learning activities, the determination and application of assessment criteria, and participation in the negotiation of social norms in the class (e.g. “I help the teacher to decide which activities are best for me”).
- The Student Negotiation (SN) dimension assesses the extent to which opportunities exist for students to explain and justify to their peers and others their newly emergent ideas and to understand and to reflect on the viability of their own and others’ viewpoints (e.g. “I talk with other students about how to solve problems”).

The constructivist view has important consequences for the development of new teaching and learning approaches that focus on students’ understanding of science and mathematics rather than recall of facts and formulas (Duit & Confrey, 1996). In mathematics, Maher and Alston (1990) proposed a long-term project that involves teachers in constructivist reform efforts. They focused on the following in describing the implications for classroom teaching: how to learn to listen to students’ thinking; how to organize classroom activities to support ‘listening and questioning’; and how to implement forms of assessment that document children’s questions. In my study, I used the revised version of CLES–Mathematics comprising the three scales of Personal Relevance (PR), Shared Control (SC), and Student Negotiation (SN) in order to study the development of a new teaching and learning approach involving systems of linear equations which focus on students’ understanding rather than recall of information. The two scales omitted were Uncertainty of Mathematics and Critical Voice because my study is not aimed at assessing dimensions that these two scales measure. Whereas Uncertainty of Mathematics assesses the extent to which students experience mathematical knowledge as coming from ever-changing human experiences and values which are culturally and socially determined, Critical Voice measures the extent to which students perceive whether it is beneficial and legitimate to question the pedagogical plans and methods which teachers use. Because my study did not have a major focus on cultural and social issues, these two CLES scales were not included.



#### 2.4.2.2 *Studies Involving Use of CLES*

Aldridge, Fraser, Taylor, and Chen (2000) used multiple research methods from different paradigms to explore the nature of classroom environments in a cross-national study involving Taiwan and Australia. Their data analysis supported the reliability and factorial validity of the CLES and revealed differences between Taiwanese and Australian classroom environments. As part of their study, multiple research methods were combined as recommended by Denzin and Lincoln (1994) and Tobin and Fraser (1998).

The CLES was translated into Chinese for use in Taiwan (Aldridge, Fraser, Taylor & Chen, 2000). In this cross-national study, the original English version was administered to 1,081 science students in 50 classes in Australia, while the new Chinese version was administered to 1,879 science students in 50 classes in Taiwan. In Taiwan, outcome-environment relationships were found for students' satisfaction in a study involving a Chinese-language version of scales of both the CLES and WIHIC (Aldridge & Fraser, 2000; Aldridge et al., 1999; Aldridge et al., 2000). The same five-factor structure emerged for the CLES in the two countries and scale reliabilities were similar.

In Singapore, a growing pool of literature that is related to classroom learning environments across different subjects includes computing (Khoo & Fraser, 1998; Teh & Fraser, 1994), geography (Chionh & Fraser, 1998), mathematics (Goh, Young, & Fraser, 1995), and science (Wong & Fraser, 1996; Wong, Young, & Fraser, 1997). In one of the studies in Singapore, Wilks (2000) expanded and modified the CLES for use among students studying English (a subject called General Paper) in junior colleges. The revised GPCLES contains two new scales called Political Awareness (reflecting Habermas' notion of emancipatory interest and assessing the extent to which students analyze causes of social injustice and advocate political reform), which are especially relevant in the teaching of General Paper. Kim, Fisher and Fraser (1999) translated the CLES into the Korean language and administered it to 1083 science students in 24 classes in 12 schools.

In other research study in Korea, outcome-environment associations have been reported for: students' attitudes to science and a Korean-language version of the

CLES, SLEI, and QTI (Lee & Fraser, 2001a, 2001b, 2002) for a sample of 440 Grade 10 and 11 science students in 13 classes; and student attitudes and Korean-language versions of CLES for a sample of 1,083 science students in 24 classes (Kim et al., 1999). The CLES has been used to study students' attitudes in Korea using a Korean-language version (Lee & Fraser, 2001a, 2001b, 2002). In Korea, Lee and Fraser (2001a, 2001b, 2002) reported the use of the CLES, SLEI, and QTI in investigating differences between streams (science-oriented, humanities-oriented) in student-perceived learning environments, while Kim et al. (1999) used the CLES in comparing the levels of perceived constructivism in Grade 10 with Grade 11.

Recently, Sebela et al. (2004) used CLES in a teacher action research in South Africa. The modified CLES was administered to a sample of 1,864 intermediate students (Grades 4–6) or senior students (Grades 7–9) students in rural, semi-rural, and urban areas in 43 classes. The use of ANOVA indicated that the modified actual form of each CLES scale was able to differentiate between the perceptions of students in different classes. Their study contributed to the modification and validation of CLES as an instrument for monitoring the development of constructivist learning environments in intermediate and senior schools in South Africa, as well as suggesting that the CLES is valid and reliable. Recent learning environment explorers who used a modified version of the CLES include: Peiro and Fraser (2005), in a study of science learning environments and student outcomes in the early childhood grades with a sample of 739 students; and Castillo, Peiro and Fraser (2005), in a study of grade-level, gender and ethnic differences in attitudes and learning environment in high-school mathematics with 600 students from 30 classes.

### **2.4.3 What Is Happening In this Class? (WIHIC) Questionnaire**

#### *2.4.3.1 Development and Description of WIHIC*

The WIHIC questionnaire brings parsimony to the field of learning environment by combining modified versions of the most salient scales from a wide range of existing questionnaires with additional scales that accommodate contemporary educational concerns (e.g., equity and constructivism) (Fraser, 1998b). The WIHIC has a separate Class form (which assesses a student's perceptions of the class as a whole)

and Personal form (which assesses a student's personal perceptions of his or her role in a classroom). The original 90-item nine-scale version was refined by statistical analysis of data from 355 middle school science students and extensive interviewing of the students concerning their views of their classroom environments in general, and the wording and salience of individual items and their questionnaire responses (Fraser, Fisher & McRobbie, 1996).

Fifty-four items in seven scales survived these procedures, although this set of items was expanded to 80 items in eight scales for the field testing of the second version of the WIHIC. Aldridge, Fraser and Huang (1999) translated WIHIC into Mandarin and validated it for use in Taiwan for a sample of 1,879 in 50 classes. After cross-validating the WIHIC with a sample of 1,081 Australian students in 50 classrooms who responded to the equivalent English version, a final form of the WIHIC containing seven eight-item scales was developed. These final modified scales are Student Cohesiveness, Teacher Support, Involvement, Investigation, Task Orientation, Cooperation, and Equity (Aldridge & Fraser, 2000).

#### *2.4.3.2 Reliability and Validity of WIHIC*

In research involving the What Is Happening In this Class? (WIHIC) questionnaire, several researchers have cross-validated the WIHIC in different countries. Dorman (2003), in an impressive study involving the use of confirmatory factor analysis, provided a convincing support for the validity of the WIHIC using a sample of 3,980 high school students from Australian, Canadian, and British schools. His study specifically employed reliability analyses, exploratory factor analysis and confirmatory factor analyses and provided substantive international validation of the WIHIC. In Canada, Raaflaub and Fraser (2002) validated the WIHIC with 1,173 students in 73 Grade 7–12 mathematics and science classrooms. The statistical analysis provided evidence of the usefulness of this instrument in describing the psychosocial factors influencing the learning environments in mathematics and science classrooms in laptop program schools in Canada.

The original version of WIHIC has been validated in Taiwan (Chen, Chang & Chang, 2002), Singapore (Fraser & Chionh, 2000), with a sample of 2,310 students in 75 senior high school mathematics and geography classes, Korea (Kim, Fisher &

Fraser, 2000), and the USA (Rickards, Bull & Fisher, 2001; Rickards, den Brok, Bull & Fisher, 2003). In a study involving the application of the WIHIC in a study of school racial diversity and socioeconomic status, Rickards et al. (2001) used a sample of 1,720 eighth-grade science students from 65 classes in 11 schools to investigate associations between school socioeconomic and racial diversity factors and students' perceptions of their science classroom learning environments.

In addition to the above studies, the WIHIC has been validated also in Indonesia using a sample of 2,498 university students in 50 computing classes (Margianti, Fraser & Aldridge, 2001a, 2002) and 422 students in 12 research methods classes (Soerjaningsih, Fraser & Aldridge, 2001a). Recently, in India, Koul and Fisher (2005) validated the WIHIC with a sample of 1,021 Grades 9–10 students in 31 classes from seven coeducational schools. In another study, Zandvliet and Fraser (2004, 2005) administered five scales selected and adapted from WIHIC to investigate the use of internet technology in high school classrooms in Australia and Canada. Their study supported the reliability and validity of the WIHIC when used in internet technology-based classrooms, and that it measured distinct, though somewhat overlapping, aspects of psychosocial environment.

#### 2.4.3.3 *Studies Involving WIHIC*

Several researchers have used the WIHIC instrument in recent studies. Many of these studies have combined the WIHIC with other scales in studying classroom learning environment (e.g. Aldridge et al., 1999, Aldridge & Fraser, 2000; Dorman, 2003; Zandvliet & Fraser, 2005).

Other recent studies have involved the use of classroom environment inventories in cross-national studies. Aldridge et al. (1999) and Aldridge and Fraser (2000), in a cross-national study involving six Australian and seven Taiwanese researchers, administered the WIHIC to 50 junior high school science classes in Taiwan (1,879 students) and Australia (1,081 students) to explore the nature of classroom environment in these countries. Although the differences were small, Australian students consistently perceived their learning environments more favorably than the Taiwanese students. In another cross-national study, Dorman (2003) studied the structural attributes of the WIHIC with a large sample of 3,980 students from

Australian, Canadian, and British high schools. His study showed the WIHIC to be a valid measure of classroom psychosocial environment. Zandvliet and Fraser (2005) reported an investigation of the use of internet technologies in high school classrooms in Australia and Canada. They specifically combined studies of the physical and psychosocial learning environments featured within these 'technological settings' and investigated interactions among the selected physical and psychosocial factors influencing students' satisfaction with their learning in these settings. The results suggest that increasing the number of computers in a setting can be counterproductive as far as maximizing students' classroom involvement is concerned. Also, their findings highlight that the arrangement of computers in the classroom influences the interaction amongst students.

Dorman (2001) conducted research into associations between classroom psychosocial environment and academic efficacy using a sample of 1,055 mathematics students from Australian secondary schools who responded to the CLES and WIHIC. Using simple correlation and multiple regression analyses, it was revealed that there was a statistically significant relationship between these classroom environment dimensions and academic efficacy (Dorman 2001). Similarly, Chen et al. (2002) utilized the WIHIC to study gender differences in students' perceptions of classroom climate in a trial of an interdisciplinary teacher-development module in Taiwan. In the USA, researchers have recently employed the WIHIC in classroom environment studies with a variety of purposes (see Chapman & Fraser, 2005; Hardy-Deveux & Fraser, 2005; Rickards et al., 2001, 2003).

Margianti et al. (2002) examined university students' perceptions of learning environment and investigated the relationship between environment and students' outcomes using a sample of 2,498 students in a private university in Indonesia. Raaflaub and Fraser (2002) used the WIHIC questionnaire to describe and compare students' perceptions of their actual and preferred learning environments in Canadian mathematics and science classrooms. Fraser (1998b, 2002) reported that the WIHIC has been used successfully in its original form or in a modified form in studies involving 250 adult learners in Singapore (Khoo & Fraser, 1997), 2,310 high school students in Singapore (Chionh & Fraser 1998), 644 high school students in 35 chemistry classes in Brunei (Riah & Fraser, 1998, 1999), 364 students in the United

States (Moss & Fraser, 2001), and 1,055 students drawn from nine Australian secondary schools (Dorman, 2001).

## **2.5 PAST RESEARCH ON LEARNING ENVIRONMENTS**

According to Fraser (1998a, 1998b), there are over 12 lines of past research on classroom learning environment that include (1) associations between student outcomes and the learning environment (Dorman, 2001; Majeed et al., 2002; Teh & Fraser, 1995a), (2) evaluating educational innovations (Khoo & Fraser, 1997; Mink & Fraser, 2005; Spinner & Fraser, 2005), (3) differences between student and teacher perceptions (Fisher & Fraser, 1983a; Maor & Fraser, 1996), (4) whether students achieve better in their preferred environment (Dart et al., 1999; Fraser & Fisher, 1983b), (5) teachers' use of learning environment perceptions in guiding improvements in classrooms (Sebela et al., 2004; Yarrow et al., 1997), (6) teacher action research (Dart, Burnett, Boulton-Lewis, Campbell, Smith, & McCrindle, 1999; Harwell et al., 2001; Sebela et al., 2004), (7) combining quantitative and qualitative methods (Fraser & Tobin, 1991; Spinner & Fraser, 2005; Tobin & Fraser, 1998), (8) links between different educational environments such as the home and the school (Marjoribanks, 1991; Moos, 1979), (9) cross-national studies (Dorman, 2003; Aldridge et al., 1999), (10) the transition from primary to high school (Ferguson & Fraser, 1999), and incorporating learning environment ideas into (11) school psychology (Burden & Fraser, 1993) and (12) teacher education (Martin-Dunlop, 2003, 2005).

My study involved the use of learning environment dimensions in evaluating an innovative strategy, as well as an investigation of associations between the learning environment and student outcomes. Because these two lines of past research are centrally relevant to my study, Sections 2.5.1 and 2.5.2 are devoted to detailing literature reviews on:

- Evaluation of educational innovations (Section 2.5.1)
- Outcome-environment associations (Section 2.5.2).

### 2.5.1 Evaluation of Educational Innovations

Classroom environment instruments have been used extensively in the evaluation of educational innovations. Past studies have used learning environment perceptions in evaluating: innovative mathematics programs (e.g. Raaflaub & Fraser, 2002; Spinner & Fraser, 2005); technology integration in the curriculum (Raaflaub & Fraser, 2002; Harwell et al., 2001; Zandvliet & Fraser, 2005); an integrated science learning environment (Nix et al., 2005); inquiry-based computer-assisted learning (Maor & Fraser, 1996); computer-assisted learning (Teh & Fraser, 1994); and a K–5 mathematics program which integrates children’s literature (Mink & Fraser, 2005).

Spinner and Fraser (2005) reported that dull classroom environments, poor students’ attitudes, and inhibited conceptual development led to the creation of an innovative mathematics program, the *Class Banking System* (CBS), which enables teachers to use constructivist ideas and approaches. They found that a comparison of CBS students with non-CBS students suggested that CBS students experienced more favorable changes in terms of a mathematics concept development, attitudes to mathematics, and perceived classroom environments on several dimensions of ICEQ, CLES, and TOMRA. Because their small sample of 119 fifth-grade students lacked the statistical power needed to adequately analyze and interpret their data, they did not perform ANCOVA to show that the gain made by the CBS students was not a result of pretest conditions. In similar research, Mink and Fraser (2005) described a one-year study of 120 fifth-grade students whose teachers participated in the Science and Mathematics Integrated with Literary Experiences (SMILE) program. SMILE was evaluated in terms of whether its classroom implementation positively influenced the classroom environment and student attitudes toward reading, writing, and mathematics. They reported that the program was successful in promoting students’ positive attitudes towards mathematics and positive changes in classroom environment.

In evaluating an integrated science learning environment, Nix, Fraser, and Ledbetter (2005) reported the impact of an innovative teacher development program based on the Integrated Science Learning Environment (ISLE) model in school classrooms. They found that students whose science teachers had attended the ISLE program

perceived higher levels of personal relevance and uncertainty of science in their classrooms relative to the classrooms of other science teachers in the same school.

Harwell et al. (2001) described action research involving using learning environments to monitor alignment of classroom learning activities and technology integration in the classroom with a constructivist viewpoint. The study yielded no significant changes in student perceptions of the classroom learning environment over the duration of the academic year. Notwithstanding, their research led the teachers to construct a new plan of action to bring their classroom learning environment into closer alignment with a constructivist perspective for teaching and learning.

In their study involving the use of classroom environment perceptions in evaluating inquiry-based computer-assisted learning, Maor and Fraser (1996) found that there was an increase in student-perceived investigation and open-endedness after using a computer database which has the potential for promoting inquiry skills. Although teachers' and students' perceptions showed a similar trend, teachers' perceptions generally were more positive than those of the students. Also, Teh and Fraser's (1994) study of an evaluation of computer-assisted learning in terms of achievement, attitudes, and classroom environment showed that massive effect sizes of 3.5 for achievement and 1.4 for attitudes were revealed when a micro-PROLOG-based CAL was used. They reported that, based on effect sizes, CAL students perceived their classes as having greater gender equity, investigation, innovation, and resource adequacy.

Currently, several researchers are using classroom learning environment in the evaluation of different educational programs. Some of these new explorations of classroom environment studies are in the United States. Hardy-Deveux and Fraser (2005), for example, recently reported an ongoing evaluation of the use of portfolios in promoting improvements in mathematics. Similarly, Chapman and Fraser (2005) reported ongoing research which evaluated the use of the exchange-of-knowledge method among Grade 5–8 mathematics students in Georgia.



### 2.5.2 Outcome-Environment Associations

Associations between classroom learning environment and students' cognitive and attitudinal learning outcomes have been studied extensively. Learning environment research often has involved investigating associations between students' cognitive and affective learning outcomes and their perceptions of psychosocial aspects of their learning environment (Dart et al., 1999; Fraser 1998a; Fraser & Chionh, 2000; Fraser & McRobbie, 1993; Margianti, Fraser & Aldridge, 2001a, 2002; Spinner & Fraser, 2005). In studies in mathematics classrooms, Dorman (2001), Majeed et al. (2002), Raaflaub and Fraser (2002), and Sebela et al. (2004) investigated associations between learning environment and attitudes to mathematics and achievement. These studies showed that there are associations between learning environment and achievement and attitudes. Dart et al. (1999), in particular, investigated the relationship between perceptions of the classroom learning environment, approaches to learning, and self concept. They showed that deep approaches to learning were perceived to be highly personalized and encourage active participation in the learning process and the use of investigative skills in learning activities. High learner self-concept scores were positively associated with deep approaches to learning and with classrooms perceived as high in personalization (Dart et al, 1999). The classroom in question was negatively associated with surface approaches to learning as expected.

Dorman (2001) investigated associations between psychosocial environment and academic efficacy. He showed that classroom environment relates positively with academic efficacy. Using commonality analysis, Dorman also showed that three scales from the CLES did not contribute greatly to explaining variance in academic efficacy beyond that attributed to the seven scales in the WIHIC. On the other hand, Sebela et al. (2004) determined whether relationships exist between students' perceptions of the learning environment and their satisfaction with their mathematics classes. They suggested that improved student attitudes are associated with more emphasis on all of the aspects of constructivism assessed by CLES. Margianti, Fraser and Aldridge (2002) described a study of learning environment factors that could influence outcomes among university students.

Majeed et al. (2002) reported a study of lower secondary mathematics classroom learning environments in Brunei Darussalam and its association with students' satisfaction. The study revealed statistically significant associations between satisfaction and the learning environment for most MCI scales. Recently, Hardy-Deveux and Fraser's (2005) study involving associations between student outcomes and classroom environment yielded nonsignificant results. In general, most recent studies have shown that there are associations between learning environment and attitudes (e.g. Chapman & Fraser, 2005; Peiro & Fraser, 2005; Raaflaub & Fraser, 2002; Santiboon, 2005; Zandvliet & Fraser, 2005). These studies have shown that students' perception of their learning environment account for appreciable amount of variance in learning outcomes (Fraser, 1994).

## **2.6 ASSESSMENT OF ATTITUDES TO MATHEMATICS**

Past and current studies have investigated students' attitude to learning (e.g. Chapman & Fraser, 2005; Hardy-Deveux & Fraser, 2005; Mink & Fraser, 2005; Raafluab, 2002; Sebela et al., 2004; Spinner & Fraser, 2005; Zandvliet & Fraser, 2005). These studies used different assessment methods to measure students' attitudes towards their computer, science and mathematics classrooms. Raaflaub and Fraser (2002), for example, assessed students' attitudes towards science using the Test of Science-Related Attitudes TOSRA (Fraser, 1981a) and attitudes towards using computers using eight items based on the Computer Attitudes Survey (CAS). They found that, relative to mathematics classes, science classes had statistically significantly higher scores on attitudes towards the subject. In Mink and Fraser's (2005) study of *Project SMILE* using a survey based on the 1988 NAEP attitude survey, reported that the program was successful with students in K–5 elementary classes in terms of promoting students' positive attitudes towards mathematics. Sebela et al. (2004) and Spinner and Fraser (2005), using the CLES and attitude scales adopted from TOSRA, found that student attitudes were associated with more emphasis on CLES dimensions.

In my study, an innovative strategy for teaching and learning systems of linear equations was assessed partly in terms of students' attitudes as assessed using an attitude questionnaire. To assess students' attitudes towards mathematics in my

study, modified scales of the Test of Science-Related Attitudes (TOSRA) (Fraser, 1981a) were used to form the Test of Mathematics-Related Attitudes (TOMRA). For instance, the item “Science lessons are fun” was changed to “Mathematics lessons are fun”. Other researchers (e.g. Chapman & Fraser, 2005; Spinner & Fraser, 2005) have adapted two of the original TOSRA scales with six items in each scale to assess student attitudes to mathematics. The scales chosen by Spinner and Fraser were Normality of Mathematicians and Enjoyment of Mathematics Lessons, while Chapman and Fraser used Inquiry and Enjoyment of Mathematics Lessons scales. TOMRA is beginning to be used widely in classroom attitude studies.

Modifying the Test of Science-Related Attitudes (TOSRA) questionnaire to obtain information about students’ attitudes toward different subjects has been reported (Chapman & Fraser, 2005; Sebela et al., 2004; Spinner & Fraser, 2005). Attitudes towards the subjects of mathematics and science have been assessed based on the TOSRA (Fraser, 1981a) by different researchers (e.g. Raaflaub & Fraser, 2002). Raaflaub used six items to study students’ attitude toward science and mathematics. The six items that they used measure the extent to which students enjoy, are interested in and look forward to science lessons. An example of an attitude item is “I really enjoy going to science class”. The wording was changed for mathematics classes to read “I really enjoy going to mathematics class”. All items selected were scored positively (Raaflaub & Fraser, 2002). Zandvliet and Fraser (2004, 2005) also modified TOSRA to assess students’ satisfaction with their learning environment. To investigate associations between the classroom learning environment and student attitude, Sebela et al. (2004) adapted a scale from TOSRA and administered it to assess student attitudes towards their mathematics classrooms.

In my research, two scales of TOMRA were administered to gather information about changes in mathematics attitudes during the use of an innovative strategy to teach systems of linear equations. In my study, the scales of Normality of Mathematicians and Enjoyment of Mathematics Lessons were administered as a pretest and a posttest as recommended by Fraser (1981a) to obtain information about changes in mathematics attitudes.

## 2.7 CONCEPT MAPS

My study incorporated both the qualitative and quantitative methods. As part of the qualitative information-gathering techniques used in this study, a concept map was utilized to obtain information regarding students' concept change as they learn systems of linear equations and to gain insight into students' concept development. It was used also as an 'advance organizer' for learning in my classroom. Concept mapping was originated in 1972 from a research program that required a way to represent changes in the knowledge structures of students who have experienced schooling for over 12 years (Novak & Musonda, 1991). The technique of concept mapping was developed and has been found to be useful in a variety of applications, including helping students to 'learn how to learn' (Novak & Gowin, 1984). Concept mapping is an effective 'advance organizer' for learning in the classroom (Kankkunen, 2001). One of its key strengths, Kankkunen wrote, is how it helps the teacher track students' conceptual development in relation to the curriculum. Another is how it keeps the teacher focused on the process of meaning-making. Novak (1996) stated that concept mapping is rooted in a constructivist epistemology that assumes that human beings construct meanings for events and objects that occur in their experience. They defined 'concept' as a perceived regularity in events or objects designed by a label (usually a word). Concept maps serve to show relationships between concepts, and it is from these relationships that concepts derive their meaning. In my study, I used concept maps to obtain information regarding the students' understanding of the solution sequence of system of linear equations (SLE). Figure 2.1 is an example of concept map for SLE. In Figure 2.1, I show the comprehensive solution sequence of learning and teaching of system of linear equations at all levels using a concept map.



Figure 2.1. Concept Map for Solving Systems of Linear Equations

Novak (1996) claims that, although concept mapping remains useful as a research tool to represent knowledge structure, it has been found to be useful in a variety of applications, including facilitation of meaningful learning, design of instructional materials, identification of misconceptions or alternative conceptions, evaluation of learning, facilitation of cooperative learning, and encouragement of teachers and

students to understand the constructed nature of knowledge (Novak, 1990a, 1990b; Novak & Wandersee, 1990 cited in Treagust et al., 1996). Sharan (1980) and Slavin (1987) claim that concept mapping can be a powerful tool to facilitate meaning making and to facilitate a sense of personal control over meaning-making for future citizens.

The rationale for utilizing concept maps in my research was to enable me to identify misconceptions or alternative conceptions that students might have or develop while learning about systems of linear equations, to graphically organize information, and to evaluate achievement. In a recent study, Kankkunen (2001) applied the method of concept mapping and interpreted it in the light of the semiotic paradigm. The success of using concept mapping in attempting to create a conceptually-meaningful learning environment was evaluated. It was shown that that concept mapping provided a means for students to discover tentative meanings for the concepts taught based on qualitative evidence (Kankkunen, 2001). Spinner and Fraser (2005) used concept maps to collect data for their research. The students' concept maps showed their understanding of the relationships among mathematics concepts, and it is from those relationships that the students' concepts derived their meaning. When students were asked to construct their concept maps, they move from patterns of rote learning to patterns of meaningful learning. Concept maps were used for identifying students' misconceptions, fostering conceptual understandings, evaluating, and therefore improving conceptual development requiring high levels of synthesis and evaluations (Feldsine, 1983).

## **2.8 COMPUTER APPLICATIONS IN TEACHING AND LEARNING SYSTEMS OF LINEAR EQUATIONS**

The argument that technology has the potential to help the students to attain mastery (Bergen, 2003) of algebra concept motivated its use in my study. Hence computer applications in teaching and learning systems of linear equations are reviewed in this section. There is no question that computer software focused on reinforcing basic skills has been popular with teachers; these programs provide enjoyable ways for children to practice skills that need to be repeated in order to be mastered (Bergen, 2003). In order to help students to improve their cognitive ability, lessons were

designed during my study so that the students could use technology to solve problems rather than listen to lectures. This meant that all instruction could take place in a computer laboratory so that the students could be working with computers continuously during class time (Urban-Lurain, 2001).

As part of my study, the experimental group students used technology to reinforce the concept that they learned in order for them to attain mastery of the skills expected. Technology can foster conjecturing, justification, and generalization by enabling fast, accurate computation, collection and analysis of data, and exploration of multiple representational forms (Goos, Galbraith, Renshaw, & Geiger, 2003). Goos et al. (2003) suggested that cognitive reorganization occurs when learners' interaction with technology as a new semiotic system qualitatively transforms their thinking. Chiappini and Bottino (2002) claimed that visual imagery takes place in mathematics learning and that there is a dialectic developed between dynamic external visual representations mediated by the technology (information visualization) and visual imagery. Therefore, software programs such as *Understanding Mathematics* and activities that focus on problem solving and concept construction skills that help students to progress independently through levels of ability could be useful if integrated in the lesson.

The software that the students used in my study was *Understanding Mathematics* (UMath). *Understanding Mathematics* software, version 2002 is a computer learning tool designed by Neufeldmath.com which emphasizes:

- understanding, not memorizing
- learning from concrete to abstract
- thinking and doing, rather than mimicking mathematics
- that a mistake is an opportunity to learn.

The main menu topics are organized into:

- a concept section
- an example questions section

- a topic test section.

I decided to incorporate this program in my study based on its:

- user-friendly nature of the program
- emphasis on understanding not memorizing
- emphasis on learning from concrete to abstract and thinking and doing, rather than mimicking mathematics,
- popularity with my previous students and their preference for this software over other similar software.

In designing multimedia systems, Sanchez, Encinas, Fernandez, and Sanchez (2002) suggested that such systems improve information-storing capacity and offer greater possibilities of adaptation and simulation, thus bringing the user closer to actual manipulation and to concrete experiences so that the student can direct learning. This is what the use of *Understanding Mathematics* software provides to the students. According to Sanchez et al. (2002), students must understand the process of solving problems in which the student calculates, conjectures, and suggests explanations, all in order to reach a solution to a problem posed. *Understanding Mathematics* software provides an opportunity for students and users to make mistakes in order to learn. The capacity and ability to solve problems is not only acquired by solving many problems, but also by acquiring ease and familiarity with different solving techniques and by discovering the mental processes used in solving one of them (Sanchez et al., p. 307). They argued that deep learning is not constructed linearly, but rather by forming propositional networks with nodes connected to each other by many cross-links of different levels. This is what the textbook could not offer in that the learning it provides is linear.

Continuing research in the field of computer application in teaching and learning suggests that the successful use of computers means involving students and educators in the learning process in new way (Zandvliet & Fraser, 2004, 2005). Therefore, much technological change is occurring in schools around the world and parents and educators alike have increasingly looked at new information and communications technology as a technical aid in developing new models for teaching and learning



(Zandvliet & Fraser, 2004, 2005). As the usefulness of applying technology and computers in teaching and learning expands, several studies involving investigation of computer-assisted learning environments (e.g. Khoo & Fraser, 1997; Maor & Fraser, 1996; Newby & Fisher, 1997; Pelton & Pelton, 2005; Teh & Fraser, 1994; Zandvliet & Fraser, 2004, 2005) have been completed.

## **2.9 TEACHER-AS-RESEARCHER**

In my study involving the evaluation of an innovative strategy for teaching and learning systems of linear equations in terms of classroom learning environment, attitudes, and conceptual development, I was a teacher-as-researcher. Therefore, it is important that I review literature on teacher-as-researcher in this section.

Often in discussions of teacher research, ‘teacher research’ and ‘action research’ are used interchangeably (Garin, 2005). Garin defined teachers-as-researchers as any preservice or inservice teachers participating in action research, inquiry groups or study groups. Garin explained that these teacher researchers could be involved in school district-sponsored professional development opportunities or university-sponsored opportunities through school-university partnerships professional development.

There are other definitions and explanations of teachers-as-researchers. Some of these definitions hinge on action research, inquiry group, and study group, and are components of teacher research. Kemmis and McTaggart’s (1988) definition emphasizes an action research cycle that builds on teacher reflection and offers the opportunity to change or amend research questions. This is an important and often overlooked skill for teacher researchers. Therefore, action research is the most formal type of teacher research. An inquiry group is less formal and provides teachers with real intellectual discourse and investigation tied to the particulars of teaching practices and new ways for teacher to interact (Garin, 2005). Murphy (1998) defined a study group as a small number of individuals joining together to increase their capacities through new learning for the benefit of students.

Therefore, the benefit of teacher-as-researchers is that teachers participating in teacher inquiry become more collaborative and collegial and more expert in their content knowledge, which leads to professional confidence and teacher efficacy (Garin, 2003; Hubbard & Power, 1999; Little, 1984). As reported by Garin (2005), additional time is identified as the number one support identified by teachers researchers, followed by teachers knowing that they can implement the results of their research, conduct research with others, and being able to select the focus of their research. Lack of time is the primary obstacle that teacher-researchers face, followed by not being permitted to identify a research topic and not being able to implement the results of the teacher inquiry (Garin, 2005).

## **2.10 CONCLUSION**

The literature relevant to this study of learning environment, students' attitudes to mathematics, and conceptual development was reviewed in this chapter. This included a review of classroom learning environment from a theoretical and historical perspective. In particular, emphasis was placed on the specific learning environments questionnaires utilized in my study. Also I reviewed the development, validation, and application of other well-known classroom learning environment questionnaires. In order to clarify the specific learning environment questionnaires used in this study, a review of literature on the development, salient features, reliability, and validity of the Constructivist Learning Environment Survey (CLES) and What Is Happening In this Class? (WIHIC) questionnaire was included. To assess students' attitudes towards mathematics in my study, the Test of Science-Related Attitudes (TOSRA) (Fraser, 1981a) was modified to form the Test of Mathematics-Related Attitudes (TOMRA). Therefore, a review of literature on attitude assessment and the salient features of the TOMRA was also included. Also in Chapter 2, literature on qualitative and quantitative research methods was reviewed. The qualitative data-gathering methods utilized in my study were audiotaped interview, students' reflective journals, students' work samples, and concept maps.

A brief overview of a range of past lines of research was included, especially the use of learning environment dimensions as criteria in evaluating educational programs

and in investigations of outcome-environment associations, because they were centrally relevant to my study. In my study, I was a teacher-as-researcher. Therefore, a review of literature on teacher-as-researcher was included in this chapter. In summary, Chapter 2 focused on reviewing literature according to the following headings:

- Theoretical and historical background of learning environment (Section 2.2)
- Qualitative and quantitative research methods (Section 2.3)
- Learning environment questionnaires (Section 2.4)
- Past research on learning environments (Section 2.5)
- Assessment of attitudes to mathematics (Section 2.6)
- Concept maps (Section 2.7)
- Computer applications for teaching and learning SLE (Section 2.8)
- Teacher-as-researcher (Section 2.9).

Whereas this chapter comprehensively provided a review of literature relevant to the study, the next chapter presents the research methodology utilized in this study.

## **Chapter 3**

### **RESEARCH METHODOLOGY**

#### **3.1 OVERVIEW**

My study evaluated an innovative strategy for teaching and learning of systems of linear equations in terms of classroom learning environment, students' attitudes to mathematics and conceptual development in California middle schools. In this chapter, therefore, the research methodology is the main focus. Data-collection processes, data sources, and the research paradigm are presented. The data types used (quantitative and qualitative), coding, and the purpose of the case study, (journals and interviews) are presented. Achievement testing and student journal writing, which are some of the tools used to gather information regarding the students' conceptual understanding of systems of linear equations, are included in this chapter.

The research methodology used in this study is based on a research paradigm previously used in similar studies. Previous studies involving classroom learning environment have used three common approaches: systematic observation, case studies, and assessing student and teacher perceptions. In designing this study, both the data sources and methods of analysis were triangulated (Guba & Lincoln, 1985). The approach utilized in the evaluation of the innovative strategy for teaching and learning of systems of linear equations is similar to that of Moschkovich (1999), which hinges on hypothesis development during classroom observations, verification and refinement through analysis of written and audiotaped data. Both quantitative and qualitative data were obtained using classroom environment questionnaires (Aldridge, Fraser & Huang, 1999; Blose & Fisher, 2003; Dorman, Fraser & McRobbie, 1994; Harwell et al., 2001; Maor & Fraser, 1996), an attitude test (Fraser, 1981a; Raaflaub & Fraser, 2002; Sebela et al., 2004; Zandvliet, 2004), achievement tests (Larson et al, 1999), students' journal writing, and student interview (Burns, 1997; Tobin & Fraser, 1998). In addition, the teaching method used in the study is an innovative approach involving the use of a structured approach and scaffolding to teach systems of linear equations by Cramer's rule and the constructivist approach including the use of technology. These are presented in this part of the thesis.

In summary this chapter discusses the following:

- Data sources (Section 3.2)
- Role of participants (Section 3.3)
- Data collection (Section 3.4)
- Data coding and levels of statistical analysis (Section 3.5)
- Data analysis and interpretation (Section 3.6)
- Lesson delivery (Section 3.7)
- Limitations of the research (Section 3.8)
- Summary (Section 3.9).

## **3.2 DATA SOURCES**

In this section, the data sources including samples and ethnogeographic distribution of the students are presented. The following is an overview of this section:

- Students' ethnogeographic distribution (Section 3.2.1)
- Sample size (Section 3.2.2)
- Instructional strategy (Section 3.2.3).

Questionnaires were used to assess the classroom environment and students' attitudes to mathematics at the middle-school level in California. The effectiveness of the innovative approach was assessed in terms of classroom environment and the outcomes of attitudes, achievement and concept development among a subsample of students. The questionnaires' factorial structure, reliability, and discriminant validity, as well as gender differences in classroom environment perceptions, attitudes to mathematics, and mathematics achievement, were investigated using a sample of 661 eighth-grade students. Environment-outcomes associations were established using the same sample. However, a subsample of only 101 students was used in answering the research question about the effectiveness of the innovative method for teaching systems of linear equations in terms of classroom environment, attitudes, and conceptual development.

### **3.2.1 Students' Ethnogeographic Distribution**

The students who participated in this study were 8<sup>th</sup> grade middle-school students from a low socio-economic community. The ethnogeographic distribution of the students is about 46% African American, 51% Hispanic and 3% other. The students' ages range between 13 and 14 years. The classes were heterogeneous in terms of previous mathematics achievement scores in the California Standardized Test (CST).

### **3.2.2 Sample Size**

A total of 661 students responded to the learning environment and attitude surveys. These data were used to answer research questions about the validity of the instruments and about associations between environment and attitudes among middle-school students in California. These questionnaires were administered by six other teachers within the same school district of which three are in the same school as the researcher, and the rest from three other middle schools. The questionnaires were administered to all students ( $N=661$ ) to collect information to answer some of the study's research questions.

Out of these 661 students, a subsample of 101 students responded to achievement and concept development questions in addition to the classroom environment and attitude survey. The decision to use this small subsample size was due to the fact that it was practicable to have this number of students in one academic year in my class. Also, these students are in my algebra class which formed four out of five period classes that I taught during the school year 2003–2004. This sample was grouped into two subgroups of 61 students and 40 students, respectively. The first subgroup formed the experimental group experiencing the innovative teaching and learning approach involving a variety of innovative learning methods. A traditional teaching approach involving the use of whole-group and direct instructional methods was used for the control subgroup of students ( $N=40$ ). Also, 12 students, comprising 8 from the experimental group and 4 from the control group, were interviewed.

### 3.2.3 Instructional Strategy

The students were exposed to a topic in algebra, such as solving systems of linear equations. A control group was taught by traditional methods, while the experimental group was taught by using the constructivist approach including computer-based instruction, simulations, and algebra software (*Understanding Mathematics* software). Scaffolding was used to help students in the experimental group to develop and form concepts and understanding for a given topic via a well-structured approach. Scaffolding instruction as a teaching strategy originates from Lev Vygotsky's socio-cultural theory and his concept of the *zone of proximal development* (ZPD). According to Vygotsky, scaffold instruction is defined as the "role of teachers and others in supporting the learner's development and providing support structures to get to that next stage or level" (Raymond, 2000, p. 176). This structured strategy involved introducing the treatment group students to the concept of matrix and the use of Cramer's rule to solve a system of linear equations. One of the objectives of this study was to investigate the effectiveness of an innovative teaching approach – involving the use of information technology, numerical methods (Cramer's rule), and constructivist methods for the topic of systems of linear equations – in terms of promoting:

- (a) a positive classroom environment
- (b) student attitudes to mathematics
- (c) student achievement and ability to identify and apply concepts.

This research question was answered through the use of classroom environment data, students' attitude data, and student conceptual development data involving 'constant comparative method' (Glaser & Strauss, 1967) across settings (experimental and control groups), individual case studies (journals and interviews), and analysis methods.

## 3.3 ROLE OF PARTICIPANTS

The role of the students who participated in this research was to follow the instructional directions and strategies and to respond to the questionnaires. The roles of the researcher and teachers included administering classroom environment and attitudes questionnaires, as well as a concept pretest before teaching the topic. The researcher also designed and delivered the instruction for the experimental group and the control group. These lessons involved the solution of a system of linear equations using graphical, substitution, and combination (elimination) methods. Once the students have mastered this topic, a numerical method involving Cramer's rule was utilized to advance conceptual understanding of the subject for the experimental group. Subsequently, the researcher administered a posttest assessing achievement of concepts. A colleague was invited to videotape the classroom during instruction.

According to Fraser (1989), despite the fact that there has been considerable classroom environment research, little progress has been made by teachers in using these ideas to guide improvement in their classroom. In my study, the teacher was the researcher. This study helped the teacher to use learning environment questionnaires to obtain feedback regarding the classroom environment and use attitude scales to gather information about the students' attitudes towards mathematics. Spinner and Fraser (2005) are among few researchers who used a similar approach – where the teacher is the researcher – to obtain feedback and conduct research in his/her own classroom. Some of the benefits of teacher-researchers using feedback are to reflect upon, discuss and question their classroom practice as a basis for improving their teaching (Elliott, 1978; Stenhouse, 1975). Teachers engaging in self-evaluation procedures should employ various feedback techniques to identify areas in which teachers' classroom behaviors differ from what they consider ideal (Bodine, 1973). Therefore, the benefit of teacher-as-researcher is shown by different researchers that teachers participating in teacher inquiry become more collaborative, collegial, and expert in their content knowledge which leads to professional confidence and teacher efficacy (Garin, 2003; Hubbard & Power, 1999; Little, 1984).

Teachers' attempts to use actual and preferred forms of classroom forms of classroom environment questionnaires to obtain feedback about, and subsequently attempt to improve their learning environments, have been reported (Fraser & Fisher,



1986). Thorp, Burden and Fraser (1994) in a study in England, Yarrow, Millwater and Fraser (1997) in a study in Australia, and Sinclair and Fraser (2002) in a study in the USA attempted to improve classroom environments as teacher-researchers. Teacher-researcher studies have the advantage in that the researcher has an unusually rich working knowledge of the research setting and close familiarity with the teaching-learning situation (Spinner & Fraser, 2005).

Nonetheless, as with all research, in addition to the strengths of this study, the constraints and disadvantages of the teacher being the researcher in this study might have affected his sound judgment and led to errors in students' assessment. The researcher in this study might not have been completely impartial during observations. Inviting a qualified mathematics teacher as an outside observer to observe the experimental group classes during the study was not always possible. If this limitation was avoidable, their feedback and critique would have been very valuable in enhancing the quality of this study. Seeing what the researcher wants to see could have obscured the sound judgment of the researcher, especially in qualitative data gathering.

### **3.4 DATA COLLECTION**

In this research, both quantitative and qualitative data-collection approaches were utilized. Fraser and Tobin (1991) illustrate the merits of combining qualitative and quantitative methods in learning environment research by drawing on three case study of successful attempts at using questionnaire surveys and ethnographic methods together within the same investigations in science education. Another merit of mixed methods research (Johnson & Onwuegbuzie, 2004) is to draw from the strengths and weaknesses of quantitative and qualitative methods in a single research studies. The use of qualitative methods in learning environment research (Tobin, Kahle, & Fraser, 1990) also has provided a more in-depth understanding of learning environments. Interpretive studies also can be enhanced with the inclusion of quantitative information (Fraser, 1998a). In interpretive research, quantitative information can be a significant component of the evidence for or against a particular assertion and the credibility of claims about patterns or relationships can be strengthened by a variety of quantitative and qualitative data sources. According to

Fraser (1998a), it is advantageous when qualitative information is complemented with quantitative information obtained from questionnaires assessing student perceptions of classroom psychosocial environment. Tobin and Fraser (1998) used multiple theoretical perspectives to frame the research and its methods, and to illustrate the desirability of combining quantitative and qualitative data to maximize the potential of research on learning environment. Aldridge, Fraser and Huang (1999) combined multiple research methods from different perspectives to examine and compare science classroom learning environments in Taiwan and Australia. They used triangulation to secure an in-depth understanding of the learning environment and to provide richness to the whole.

Quantitative information was gathered in my study using:

- three scales selected from the Constructivist Learning Environment Survey (CLES)
- three scales selected from the What Is Happening In this Class? (WIHIC) questionnaire
- two scales selected from the Test of Mathematics Related Attitudes (TOMRA)
- achievement test.

Qualitative information was collected using:

- audiotaped interview
- student journal writing
- concept map
- analysis of the students' work.

The next sections explain in depth the questionnaires used in this study including the achievement data which formed the quantitative information that was gathered for this study. Also, the qualitative information-gathering techniques (concept maps, audiotaped interview, student journal writing, students' work samples) used in this study are presented.

### **3.4.1 Constructivist Learning Environment Survey (CLES)**

A comprehensive review of literature pertaining to the Constructivist Learning Environment Survey (CLES) was presented in Section 2.4.2 in Chapter 2. In that section, the development and a description of the CLES were also provided. The underlying principle behind this measuring instrument is the constructivist view. Meaningful learning is a cognitive process in which individuals make sense of the world in relation to the knowledge which they already have constructed, and this sense-making process involves active negotiation and consensus building. The constructivist learning environment survey (CLES) (Taylor, Dawson & Fraser, 1995; Taylor, Fraser & Fisher, 1997) was developed to assist researchers and teachers to assess the degree to which a particular classroom's environment is consistent with a constructivist epistemology, and to assist teachers to reflect on their epistemological assumptions and to reshape their teaching practice. Also, the CLES can be used to assess the level of constructivist teaching and learning practices (Fraser, 1998a, pp. 534-535). Recent studies that have utilized the CLES include Nix et al. (2005), Spinner and Fraser (2005), Aldridge, Fraser, Taylor and Chen (2000), Harwell et al. (2001), Dorman (2001), and Dryden and Fraser (1996).

Dorman (2001) validated CLES with a sample of 1,055 in 27 school year groups (classes). The validation of scales reported in that study was scale internal consistency, discriminant validity, and ability to differentiate between school year groups (classes). The internal consistency ranged from 0.76 to 0.89 (student mean) and 0.82 to 0.94 (school year group mean) for the Personal Relevance, Shared Control, and Student Negotiation scales. Also, in that study, the discriminant validity was 0.36 to 0.46 (student mean) and 0.35 to 0.44 (school year group mean) for the same scales.

Aldridge, Fraser, Taylor, and Chen (2000) validated the CLES using data analysis that supported the reliability and factorial validity in a cross-national study that revealed differences between Taiwanese and Australian classroom environments. In

this study, a sample of 1,081 science students in Australia was used, while a Chinese version was administered to a sample of 1,879 science students in 50 classes in Taiwan. In other studies validating the CLES in Korea, Lee and Fraser (2001a, 2001b, 2002) used a sample of 440 grade 10 and 11 science students in 13 classes, and Kim et al. (1999) used Korean-language versions of CLES for a sample of 1,083 science students in 24 classes. These past research results replicated each other closely. Nix et al. (2005) validated CLES with a sample of 1,079 in 59 classes in the USA. They reported alpha reliability ranging from 0.74 to 0.85 (individual) and 0.85 to 0.93 (class mean) for five scales. The mean correlation with other scales ranged from 0.28 to 0.32 (individual) and 0.28 to 0.39 (class mean).

My study made use of the three CLES scales of Personal Relevance (measures students' perceptions of the extent to which either science or mathematics knowledge is connected to students' actual out-of-school experiences), Shared Control (assesses the extent to which students perceive that they are being invited to share control with the teacher of the total learning environment, including the design and management of learning activities, the determination and application of assessment criteria, and participation in the negotiation of social norms in the class), and Student Negotiation (assesses the extent to which opportunities exist for students to explain and justify to their peers and others their newly emergent ideas and to understand and to reflect on the viability of their own and others' viewpoints) (Appendix A). The three scales were chosen because they assess the domains within which my study is aimed at evaluating and also, these scales will not overlap with the items chosen from the WIHIC. Furthermore, the CLES has been widely validated based on discriminant validity, reliability, ability to differentiate between different classes, and factor analyses. The CLES was chosen to be used in my study to evaluate the innovative teaching and learning approach which is fashioned after the constructivist learning approach. It was anticipated that an underlying shift from teacher-centered instruction to learner-centered construction of knowledge would occur with this innovation (Taylor & Fisher, 1991; Dorman, 2001; Harwell et al., 2001), which included technology, innovation in learning systems of linear equations, and the application of Cramer's rule. The Actual version of the Constructivist Learning Environment Survey (CLES) was administered to all students prior to the lessons and

after the lessons. This helped the researcher to determine the degree to which the classroom environments were consistent with a constructivist epistemology.

### **3.4.2 What Is Happening In this Class? (WIHIC) Questionnaire**

In Chapter 2, the What Is Happening In this Class? (WIHIC) questionnaire has been reviewed including the development and description of the questionnaire. As mentioned in Section 2.4.3, the WIHIC questionnaire brings parsimony to the field of learning environments by combining modified versions of the most salient scales from a wide range of existing questionnaires with additional scales that accommodate contemporary educational concerns (e.g., equity and constructivism) (Fraser, 1998b). According to Fraser, the WIHIC has been used successfully in its original form or in a modified form in studies involving 250 adult learners in Singapore (Khoo & Fraser, 1997), 2,310 high school students in Singapore (Chionh & Fraser, 1998), and 1,055 students drawn from nine Australian secondary schools (Dorman, 2001). Chen, Chang and Chang (2002) used the WIHIC in their study to collect information regarding student's perceptions of the curriculum innovation in order to determine students' perceptions after the curriculum change.

This instrument was selected not only because it has been validated widely, but also because it combines the most salient scales from a wide range of existing questionnaires. It also accommodates modern educational emphasis by including equity and constructivism concerns. My choice of WIHIC for this study is also based on the finding that it is capable of differentiating between the perceptions of students in different classrooms (Rickards, Bull & Fisher, 2001). The WIHIC enabled me to differentiate between the perceptions of students in the experimental group and the control group. While Rickards et al. (2001) attempted to employ the WIHIC within schools whose populations are defined as having high, medium, or low diversities of race (such as the one captured in this study) and what perceptions those student have of their science classes, this study tried to capture this situation within their mathematics classes.

In this study, I used three scales from the WIHIC (Appendix B) to collect data. These scales are Involvement (the extent to which students have attentive interest,

participate in discussions, do additional work and enjoy the class), Investigation (the extent to which skills and processes of inquiry and their use in problem solving and investigation are emphasized), and Task Orientation (the extent to which it is important to complete activities planned and to stay on the subject matter). The reason for using these three scales was to avoid the overlapping of items from the CLES which assessed the salient points that are not included in the WIHIC. Several researchers have cited similar reasons for using few scales based on reliability and discriminant analysis and overlapping of some items in past studies (e.g. Raaflaub & Fraser, 2002; Sebela et al. 2004; Spinner & Fraser, 2005).

### **3.4.3 Test of Mathematics-Related Attitudes (TOMRA)**

Few studies have been reported involving the use of TOMRA, although this questionnaire was adapted from the widely-used Test of Science Related Attitude (TOSRA, Fraser, 1981a). Studies that involve modifying the Test of Science-Related Attitudes (TOSRA) questionnaire to obtain information about students' attitudes toward different subjects have been reported. Students' attitudes towards mathematics and science based on the TOSRA (Fraser, 1981a) have been investigated by Raaflaub and Fraser (2002). Raaflaub and Fraser used six items to study students' attitudes towards science and mathematics in terms of the extent to which students enjoy, are interested in, and look forward to lessons. Zandvliet & Fraser (2004, 2005) also modified and included TOSRA in assessing students' satisfaction with their learning. Also, Mink and Fraser (2005) used TOMRA for mathematics classes. To investigate associations between the classroom learning environment and student attitudes, Sebela et al. (2004) administered a scale adapted from the TOSRA to assess student attitudes towards their mathematics classrooms.

These researchers modified and incorporated the TOSRA in their studies to assess the students' attitudes towards their various subjects and classrooms. In order to assess students' attitudes towards mathematics, the Test of Science-Related Attitudes (TOSRA) has been modified to form the Test of Mathematics-Related Attitudes (TOMRA) (Spinner & Fraser, 2005). For instance, the item "Science lessons are fun" was changed to "Mathematics lessons are fun." Therefore, in my research, two scales of TOMRA were administered to gather information about changes in

mathematics attitudes during the use of innovative strategies to teach systems of linear equations. The scales chosen were Normality of Mathematicians and Enjoyment of Mathematics Lessons (Appendix C). TOMRA was administered as a pretest and a posttest as recommended by Fraser (1981a) to obtain information in my study about changes in mathematics attitudes.

#### **3.4.4 Achievement Test**

In order to evaluate the effectiveness of the innovative teaching and learning approach in terms of student achievement, I administered an achievement pretest and a posttest involving the concepts of systems of linear equations learned. These tests were used to evaluate whether the students learned the new concepts. Also, these tests were used to compare the relative effectiveness of the innovative approach and the traditional approach for the learning of systems of linear equations. Spinner and Fraser (2005) used achievement tests to evaluate how well the students learned the Class Banking program in the elementary classroom. Student responses on the pre- and post-assessments were used to document the extent of student conceptions and to assess any changes in student conceptions after the lessons (Moschkovich, 1999). The achievement test can give clearer insight into how the students learned a new concept. The outcome of the test was used to make a decision regarding the extent to which the students understood the concept of systems of linear equations and whether the innovative approach is a better teaching method for the topic of systems of linear equations.

Five open-ended questions, as recommended by Treagust, Jacobowitz, Gallagher, and Parker (2003) and used by Wong (1993, 1996), were given to the students in order to measure their prior knowledge of the concepts of linear equations and systems of linear equations. This approach helped the researcher to identify students' personal conceptions, misconceptions, and problems in understanding the topic (Treagust et al., 2003). The open-ended questions administered for the pretest are shown in Appendix D. The same set of questions was given to the students at the end of the third week of instruction on the topic of systems of linear equations with the exception of Question 4 which was modified. The modification to Question # 4 involved a different problem relating to real-life application of systems of linear

equations – with which the students targeted for this study could easily identify – was used to replace the question (see Appendix D).

Another test consisting of 10 multiple-choice questions on the topic of systems of linear equations was also administered at the end of the fourth week of instruction. The sample questions from which six of my items were adapted (Huson, Lundin, & Samuels, 2003) are presented in Appendix E. In order to obtain a summative performance for the students, a constructive-response test consisting four questions, including one concept map question, was administered to the experimental group at the end of the Cramer's rule method of solving systems of linear equations lesson. This assessment was used to evaluate the conceptual understanding of this group on Cramer's rule method of solving systems of linear equations. Subsequently, another multiple-choice test involving the first 10 sets of questions and another 9 sets of questions, adapted from McDougal Littell Algebra 1 Concepts and Skills (2002), was given to all the groups. These items are presented in Appendix F.

#### **3.4.5 Concept Map Test**

The technique of concept mapping was developed in 1972 by Novak and has been found to be a useful tool in a variety of applications, including helping students to 'learn how to learn' (Novak & Gowin, 1984). The rationale for utilizing a concept map in my research was to enable me to identify misconceptions or alternative conceptions that students might have or develop while learning about systems of linear equations and to evaluate achievement. The students were trained on how to use a concept map to explain their ideas and concepts of topics. In this research, the students were highly encouraged to use sequence maps to explain their thoughts about the concept especially the solution steps. Sharan (1980) and Slavin (1987) wrote that concept mapping can be a powerful tool to facilitate meaning-making and to facilitate a sense of personal control over meaning-making for future citizens.

#### **3.4.6 Student Journal Writing**

In my study, students' journal writings were collected on each of the solution methods used to teach and learn systems of linear equations. Writing assignments are



assessments that are produced by students, rather than teachers (Bush & Leinwand, 2000). They challenge students to think about their mathematical strengths and weaknesses, their attitudes, or their beliefs. They often take the form of student journals, students' reflections on their thinking and methods of working, and student-completed inventories (Bush & Leinwand).

By actively involving students in the assessment process, student writing and inventories:

- encourage students to think about how they solved a problem or performed a skill;
- help students view their accomplishments in terms of their own strengths and ideas for improvement;
- give students ways to think consciously about expanding their mathematics repertoire: type of strategies, use of representations, focus on content areas; and
- provide us with evidence that students use concepts and problem-solving strategies.

Students' journal writings were studied and also their misconceptions were exposed. This helped me to restructure the instruction in order to emphasize and change their erroneous views and conceptions about the topic. Bush and Leinwand (2000) wrote that many teachers have had success in monitoring their students' feeling and thoughts about mathematics through daily or weekly journals. Treagust et al. (2003) suggested that the use of individual writing tasks provides opportunities for each student to expand his or her personal ideas and reasoning, and reconcile them with accepted concepts and processes. Individual writing tasks can capture students' understanding so that the teacher can assess their progress (Treagust et al., 2003). Many writers have written about journal writing and the idea of promoting high-quality writing (see Ciochine & Polivka, 1997; Lou DiPillo, Sovchik & Moss, 1997; McIntosh, 1991; Norwood & Carter, 1994).

### **3.4.7 Audiotaped Interviews**

In naturalistic settings, teaching and learning events are often audiotaped or videotaped and verbatim transcripts are made of the lesson or activity (Duit et al., 1996). Their view is that field notes of the lesson or activity taken by the researcher can be written up later to provide an intensive account of the observations. Burns

(1997) employed research methods that drew on ethnographic techniques such as participant observation, interviews, survey instruments, and the collection of video and auditory records in the investigation of students' and teachers' use of technology in specific classroom environments. In my study, ethnographic techniques involving the collection of video and audiotaped interviews, were used in conjunction with survey instruments to investigate holistically students' perceptions of their classroom environment, attitudes, and understanding of the solution of systems of linear equations in two variables. When a study using quantitative methods has been completed, its main findings can be contextualized with thick descriptions consisting of observations and verbal accounts from participants (Tobin & Fraser, 1998).

### **3.5 DATA CODING AND LEVELS OF STATISTICAL ANALYSIS**

#### **3.5.1 Data Coding**

After the data-collection process was completed, the data were coded into Microsoft Excel 2001 for further analysis using SPSS version 11.5. The following steps were taken during data entry:

1. The data coding was set up for the 661 samples including the experimental group and control group. This format was set up to capture the following information: students' scores on achievement tests, students' gender, students' responses to the CLES, students' responses to the TOMRA, and students' responses to the WIHIC. The CLES, TOMRA, and WIHIC have 20, 18, and 24 columns, respectively (i.e. one column for each item). The achievement sections for experimental and control groups have 5 and 4 columns, respectively.
2. This step involved coding all students' responses to all items on the CLES, TOMRA, and WIHIC questionnaires. The coding was aimed at differentiating students into experimental group by class, control group by class, and gender. The sample ( $N=661$ ) of student responses was coded to answer the research questions. The coding was organized in this way to make analysis of the data easy.

3. This final step was aimed at putting all data together into the data format. The experimental group responses were entered first, followed by control group responses, and then the rest of the samples. Finally, this spreadsheet that contains all the data was relocated to SPSS version 11.5 for analysis process. The missing data were removed entirely except in some instances where their removal would affect the other columns. Invalid or omitted responses were scored 3 for the CLES, WIHIC and TOMRA. In the case of achievement data involving the pre-test, missing data were replaced with zero. This ensured the refinement of the data coded in order to enhance computation.

### 3.5.2 Levels of Statistical Analysis

Many past learning environment studies have employed techniques such as multiple regression analysis, but few have used multilevel analysis (Bock, 1989; Bryk & Raudenbush, 1992). This takes into account the hierarchical nature of classroom settings (Majeed, Fraser, & Aldridge, 2002). Goh et al. (1995) and Wong et al. (1997) compared the results from multiple regression analysis with those from an analysis involving hierarchical linear model. Fraser (1986a, 1994, 1998a) presented the importance of choosing an appropriate statistical analysis level or unit. By using different levels of statistical analyses, measures with the same operational definition can have different interpretations. Another reason is that there is the possibility of obtaining different results of statistical analyses when different levels of analysis are employed (Robinson, 1950). Finally, there is a possibility of introducing sampling errors (Peckham, Glass & Hopkins, 1969; Ross, 1978) when different levels of statistical analysis are applied to testing hypotheses (Burstein, Linn & Capell, 1978).

Based on these reasons, in classroom environment research design, the researcher must decide on the statistical analysis levels and, importantly, whether the study will involve the perception scores of individual students (*private beta press*) or the average scores of all the students within the same class (*consensual beta press*) (Stern, Stein, & Bloom, 1956). In my research, the perception scores of individual students (*private beta press*) and the average scores of all the students within the same class (*consensual beta press*) were both utilized as levels of statistical analysis.

### **3.6 DATA ANALYSIS AND INTERPRETATION**

SPSS (version 11.5) statistical package was used to analyze students' responses to obtain evidence for the CLES, WIHIC and TOMRA to support factor structure, scale internal consistency reliability, and the ability to differentiate between the perceptions of the learning environment among students in different classrooms. In addition, this software package was used to analyze all the quantitative data including the achievement data in order to answer the other research questions (Section 1.7) concerning the validity of questionnaires, the effectiveness of the innovative strategy in terms of promoting (a) a positive classroom environment, (b) student attitudes to mathematics, and (c) student achievement and ability to identify and apply concepts, associations between classroom environment and student attitudes to mathematics, and gender differences in classroom environment perceptions.

The following is a summary of the structure of subsections below:

- Validity and reliability of the CLES, WIHIC and TOMRA (Section 3.6.1)
- Comparison of experimental and control groups on achievement, classroom environment and attitudes (Section 3.6.2)
- Associations between students' attitudes and learning environment (Section 3.6.3)
- Gender differences in classroom environment perceptions, attitudes to mathematics and achievement (Section 3.6.4)
- Qualitative data analysis (Section 3.6.5).

#### **3.6.1 Validity and Reliability of CLES, WIHIC, and TOMRA**

A principal components factor analysis with varimax rotation was used to furnish evidence about the validity and reliability of the questionnaires for assessing classroom environment and attitudes among middle-school mathematics students in California. These factor analyses were used to determine whether all of the items from the three CLES scales in the original questionnaire (Personal Relevance, Shared

Control, and Student Negotiation) and all three items from WIHIC (Involvement, Investigation, and Task Orientation) formed independent measures of psychosocial learning environment. A principal component factor analysis with varimax rotation was also used to determine whether all of the items from the two TOMRA scales (Normality of Mathematicians and Enjoyment of Mathematics Lessons) formed two independent measures of attitudes to mathematics.

The Cronbach alpha coefficient was computed for two units of analysis (individual and class mean) for each scale of the CLES, WIHIC, and TOMRA in order to estimate the internal consistency reliability (measure of scale reliability and also to provide information about the extent of relationships between individual items in the scale). The discriminant validity (extent to which a scale measures a distinct construct that is not assessed by the other scales) of each scale was determined by calculating the mean correlation of each scale with other scales. An ANOVA was also used to determine the ability of each CLES and WIHIC scale to differentiate between the perceptions of the student in different classes. The ANOVAs yielded information about the eta-squared statistic, which is the proportion of the total variability in the dependent variable (classroom environment scores) that is accounted for by variation in the independent variable (class membership) (the ratio of the between-groups sum of squares to the total sum of squares). For each ANOVA, scores on a classroom environment scale were used as the dependent variable and class membership was the independent variable.

### **3.6.2 Comparison of Experimental and Control Groups on Achievement, Classroom Environment and Attitudes**

The pretest-posttest changes for the experimental and control groups in classroom environment perceptions, attitudes to mathematics, and mathematics achievement were analyzed using ANOVA (effect sizes and *t*-test results), ANCOVA ( $\eta^2$  and *F*-ratio), and descriptive statistics. An ANOVA analysis was performed for scores on each CLES, WIHIC, and TOMRA scale and achievement to investigate changes between pretest and posttest. Analysis of covariance (ANCOVA) was also used in order to test for differential pretest-posttest changes between the experimental and control groups at posttest. The corresponding pretest CLES, WIHIC, TOMRA or

achievement scores were used as covariate in order to control the source of variability (the covariate).

### **3.6.3 Associations between Classroom Environment and Student Outcomes**

The associations between classroom environment and student attitudes to mathematics were analyzed using simple correlation and multiple regression analyses. The sample of 661 students comprising both the experimental and control groups who participated in the study were analyzed for two units of analyses (individual and class mean).

### **3.6.4 Gender Differences in Classroom Environment Perceptions, Attitudes to Mathematics and Achievement**

A two-way MANOVA with repeated measures on one factor was utilized to explore the gender differences in terms of students' perceptions of classroom environment and attitudes to mathematics, as well as achievement on open-ended mathematics questions. The unit of analysis for comparison was the students.

### **3.6.5 Qualitative Data Analysis**

Qualitative information, gathered through audiotaped interviews, was transcribed and analyzed in order to understand the students' opinions about the new approach. Also, videotape recordings were submitted to other teachers of mathematics in order to provide the researcher with feedback about the innovative approach. Students' learning journal writing was used to obtain further information describing students' learning of the topic of systems of linear equations. Consequently, the students' work samples were analyzed to gather further information on their errors in conceptual understanding. Babbitt (1990) sorted errors into four categories: computation errors, operational errors, non-attempt errors, and miscellaneous errors, and concluded that an error analysis of student work can reveal underlying conceptual misunderstandings and a lack of appropriate strategies for problem solving. On the

other hand, Backman (1978) used four categories for procedural errors: errors in sequencing steps within a procedure, errors in selecting information or procedures, errors in recoding work, and errors in conceptual understanding. In my study I identified errors in conceptual misunderstanding and miscellaneous errors. These qualitative information-gathering tools were utilized to gather more in-depth understanding of learning environments (Tobin, Kahle, & Fraser, 1990) and insights into students' perceptions (Spinner & Fraser, 2005), students' attitudes, and mathematics achievement.

### **3.7 LESSON DESIGN AND DELIVERY**

The teaching strategies and methods used in the delivery of instruction in elementary schools have proven to be very effective in knowledge retention and mastery. In order to teach the above identified topics to middle-school students, scaffolding was used to help the students to develop and form concepts and understanding of a given topic via a well-structured approach. In addition, technology and other teaching strategies were integrated in the lessons.

Lessons were designed in order to help the students to improve their understanding of the basic concepts behind their prior misconceptions so that their misconceptions are exposed, challenged and reconciled with correct reasoning. Most of these lessons were delivered based on a general framework for teaching a mathematics topic as shown in Figure 3.

The learning of systems of linear equations was introduced using a graphical approach. This helped the students to activate their mental imagery, and hence they could relate to the visual representation afforded by the graph. This visual representation can translate to the students' mental imagery (Chiappini & Bottino, 2001; Dreyfus, 1995). The California State Mathematics Standard requires the students to have knowledge of linear graphs before coming to Grade 8. From Grade 5, the students are expected to know how to graph. Therefore, Grade Five Strand 1.4 states that students:

Identify ordered pairs of data from a graph and interpret the meaning of the data in terms of the situation depicted by the graph (*Mathematics Framework for California Public Schools*, 2000, p. 55).

Grade 7 standard 3.0 states that Students graph and interpret linear and some nonlinear functions (*Mathematics Framework for California Public Schools*, 2000, p. 67).



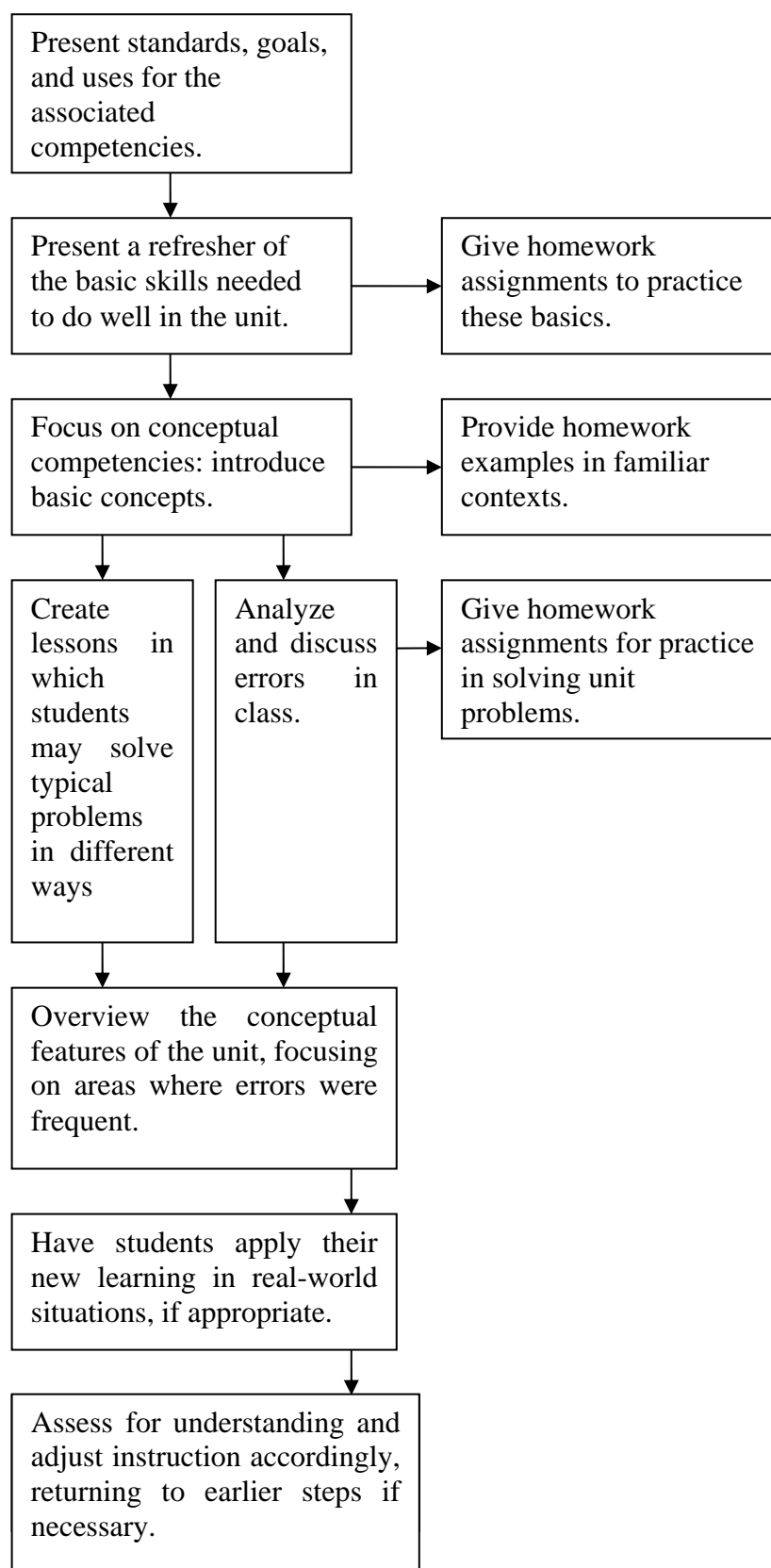


Figure 3.1. General Framework for Teaching a Mathematics Topic (*Mathematics Framework for California Public Schools*, 2000, p. 192)

Hence, the students targeted for this study were eighth grade students. An effort was made to re-teach graphing linear equations to the students who subsequently applied this concept to solving an SLE. The students were made to understand that this method of solving SLE is only an approximation in order not to build a misconception that they will face later on in the lesson. The next lesson clearly explained systems of linear equations by substitution.

The entire lesson lasted for about 10 weeks. This timeline enabled the researcher to teach the students prior skills that would help them to be successful in this topic. The prior knowledge exposure included solving linear equations in one variable, graphing linear equations, and simplifying linear equations amongst other skills. The first week of instruction involved teaching the students how to solve linear equations in one variable. The second week of instruction was used to teach the students how to graph linear equations. The third week was a lesson on graphing systems of linear equations in two variables and finding the approximate solution graphically. The fourth week of lessons focused on solving a system of linear equations in two variables using the substitution method. The fifth week of instruction involved solving systems of linear equation in two variables by the combination method. In the sixth week of instruction students explored the use of technology for solving systems of linear equations for the experimental group. The seventh week of instruction was used to introduce the concepts of matrices and matrix operations, including finding the determinant of a matrix for the experimental group. This is the skill that students need to solve SLE by Cramer's rule. The eighth and ninth weeks were used to explore solution of SLE using different methods and for posttest administration. Both the control and the experimental groups followed this plan except the lessons on Cramer's rule and exploring SLE using technology that only the experimental group experienced.

Before the start of the topic, a pretest consisting of about five open-ended questions (Appendix D) was administered to the students in order to identify personal conceptions, misconceptions, and problems in understanding the topic for both the experimental group ( $N=61$ ) and the control group ( $N=40$ ). Guided by the recommendation made by Treagust et al. (2003), the students' ideas and reasoning were elicited by asking them questions throughout each lesson (Socratic questioning

approach) to engage students and deepen their understanding about the underlying systems of linear equations principles and solution strategies. The use of journal writing and reflection were often used to help to gather enough information about the students' thinking and understanding of the concept. This is part of the requirement for the mathematics teaching standard in California (see Crowley, 1993; Kuhs, 1994; Lambdin & Walker, 1994; Stenmark, 1991). Case studies, according to Anderson and Arsenaut (1998), are holistic research methods that utilize multiple sources of evidence to analyze or evaluate specific phenomena or instance. A case study could be described as a strategy, instead of a method, which uses a system of organizing social data in such a way that the unitary nature of the social object being examined is preserved (Punch, 1998).

The teaching methodology utilized in my study involved exposing the students to prior knowledge which included solving linear equations in one variable, graphing linear equations, and simplifying linear equations amongst other topics. Therefore, *Lesson 1* shows the lesson on teaching and learning systems of linear equations by graphing. In that lesson, error analysis was explained. Subsequently, the lesson on teaching and learning systems of linear equations by substitution method was delivered including some identified misconceptions. In addition, a lesson on teaching and learning systems of linear equations by combination method was designed and delivered. The students' errors learning this solution method were identified and addressed through error analysis.

### **3.8 LIMITATIONS OF THE RESEARCH**

Threats to rigor were minimized by clearly explaining the method of answering questionnaires to respondents. Unclear and misleading data were discarded. In so doing, a high standard of rigor was attained, thereby minimizing threats to rigor. The representativeness could be another limiting factor in that, when compared to the general eighth-grade population in California, my sample could be considered neither a sizeable fraction of the population nor representative of the full range of schools and students. This limits the generalizability of findings. The statistical power could be limited, especially for the data analyses that involved the small subsample size of 101 students. Moreover, because of the smallness of the comparison of experimental

and control groups ( $N=101$ ), it was not possible to use MANOVA and MANCOVA. Therefore, using multiple  $t$ -tests and multiple ANCOVAs could have given rise to a Type I error.

The teacher being the researcher in this study could have influenced the outcomes of this study. The researcher might have been partial in observation of the students. The halo effect is a form of researcher bias that is common at the data analysis stage, which occurs when a researcher is scoring open-ended responses or the like, and allows his or her prior knowledge of or experience with the participants to influence the scores given (Onwuegbuzie & Daniel, 2003). Inviting the other teachers who teach mathematics unfortunately was not possible in all instances. It would have been advantageous for them to observe and give their opinion and constructive criticism during all the lessons involving the innovative strategy with the experimental group classes during the study. Perhaps, if this were possible, their feedback and critique would have been valuable in enhancing the quality of this study. Seeing what the researcher wants to see could have obscured the sound judgment of the researcher, especially in qualitative data gathering.

### **3.9 SUMMARY**

Chapter 3 presented the research methodology utilized in obtaining and gathering the data, organizing the data, analyzing the data, and interpreting the information for this study. In this chapter, the data sources (Section 3.2) for data-gathering were explained, including the role of the participants (Section 3.3). Also the data-collection methods (Section 3.4), data coding, and levels of statistical analysis (Section 3.5) were presented. Section 3.6 presented the statistical analysis methods used to analyze the data in order to answer the research questions, as well as the interpretation of the results. The section on lesson delivery (Section 3.7) included the lesson modules and the approach utilized in presenting the lesson during the study. The limitations of the research, minimizing threats to rigor, the restricted sample size, and the halo effect were explained in this chapter. In summary, Chapter 3 discussed the following:

- Data sources (Section 3.2)

- Role of participants (Section 3.3)
- Data collection (Section 3.4)
- Data coding and levels of statistical analysis (Section 3.5)
- Data analysis and interpretation (Section 3.6)
- Lesson delivery (Section 3.7)
- Limitations of the research (Section 3.8).

The following chapter provides the results from quantitative data analyses, a discussion of the findings, answers to the research questions, and information regarding limitations of the study.

## **Chapter 4**

### **QUANTITATIVE DATA ANALYSIS**

#### **4.1 INTRODUCTION**

This chapter is devoted to describing the data analyses and discussing the findings from the quantitative survey data and achievement tests data from this study. These findings from the quantitative data are discussed in seven sections. Each section of this chapter provides results relating to classroom environment, students' attitudes, and their conceptual development in Algebra 1.

In order to answer Research Question 1, data collected from the sample of 661 students were used to investigate for each CLES, modified WIHIC, and TOMRA scale, the factor structure, reliability, discriminant validity, and ability to distinguish between different classes and groups. The results reported in Section 4.2 below provide evidence of the validity of these instruments in describing the psychosocial factors influencing the learning environment when used in middle-school mathematics classrooms in California.

The second objective of this study was to explore whether the innovative teaching approach – involving the use of information technology, numerical methods (Cramer's rule), and constructivist methods for the topic of systems of linear equations – is effective in terms of promoting (a) a positive classroom environment, (b) student attitudes to mathematics, and (c) student achievement and student ability to identify and apply concepts. The third objective of this study was to investigate whether there are associations between classroom environment and student attitudes to mathematics. The fourth and final objective of my study was to investigate gender differences in perceptions of classroom environments, attitudes to mathematics, and achievement.

Analyses of the survey instruments and achievement tests helped to answer the following research questions:

Research Question #1

*Are questionnaires for assessing classroom environments and attitudes to mathematics valid when used with middle school students in California?*

Research Question #2

*Is an innovative teaching approach – involving the use of information technology, numerical methods (Cramer’s rule), and constructivist methods for the topic of systems of linear equations – effective in terms of promoting:*

- (a) a positive classroom environment*
- (b) student attitudes to mathematics*
- (c) student achievement and ability to identify and apply concepts?*

Research Question #3

*Are there associations between classroom environment and student attitudes to mathematics?*

Research Question # 4

*Are there gender differences in perceptions of classroom environments, attitudes to mathematics, and mathematics achievement?*

The current chapter is organized according to the following topics:

- Introduction (Section 4.1)
- Validity and reliability of CLES, WIHIC and attitude scales (Section 4.2)
- Comparison of experimental and control groups on achievement, classroom environment and attitudes (Section 4.3)
- Associations between students’ attitudes and learning environment (Section 4.4)
- Gender differences in classroom environment perceptions and attitudes to mathematics (Section 4.5)
- Limitations of my study (Section 4.6)
- Summary of the analyses and results (Section 4.7).

## 4.2 VALIDITY AND RELIABILITY OF THE CLES, WIHIC AND TOMRA

The validity and reliability of CLES, WIHIC, and TOMRA scales are reported using the following structure:

- Factor structure of CLES, WIHIC, and TOMRA (Section 4.2.1)
- Internal consistency and discriminant validity of CLES, WIHIC, and TOMRA (Section 4.2.2)
- Ability of CLES and WIHIC to differentiate between classrooms (Section 4.2.3).

### 4.2.1 Factor Structure of CLES, WIHIC, and TOMRA

Factor analysis is a data-reduction technique normally used for reducing a large number of items to a smaller set of factors or underlying variables (Coakers & Steed, 1996). A principal components factor analysis with varimax rotation with Kaiser normalization was performed separately for the CLES, WIHIC and TOMRA for the sample of 661 middle-school mathematics students to confirm the *a priori* structure. An item was retained only if its factor loading was at least 0.40 on its own scale and less than 0.40 on all other scales in that instrument.

Appendix A shows that the CLES comprises 18 items with 6 items in each of the three scales of Personal Relevance, Shared Control, and Student Negotiation. Table 4.1 shows that the factor analysis supports the original structure of the CLES as reported in the literature. Item 6 was the only question that was dropped from the CLES because its factor loading was less than 0.40 on its *a priori* scale. Item 6 is reversed scored and appeared to be problematic for the students. For each of the 17 items in Table 4.1, the factor loading was 0.40 or larger with the item's own scale and less than 0.40 with other two CLES scales.

The percentage of total variance extracted ranged from 11.18% to 27.18% for different scales and the eigenvalue associated with each factor ranged from 2.01 to



4.89. The total variance was 50.73% for the three scales of the CLES utilized in this study. The percentage variance and the eigenvalue for each scale are shown at the bottom of Tables 4.1.

Table 4.1. Factor Loadings for a Modified Version of Actual Form of the CLES in California Middle Schools

Item Number	Factor Loadings		
	Personal Relevance	Shared Control	Student Negotiation
1	0.78		
2	0.66		
3	0.40		
4	0.73		
5	0.73		
7		0.67	
8		0.69	
9		0.78	
10		0.80	
11		0.79	
12		0.64	
13			0.46
14			0.76
15			0.75
16			0.78
17			0.76
18			0.75
% Variance	11.18	27.18	12.37
Eigenvalue	2.01	4.89	2.23

*N*=661

Factor loadings smaller than 0.40 have been omitted.

A similar principal components factor analysis with varimax rotation with Kaiser normalization was also performed for the same sample of 661 middle-school mathematics students to confirm the *a priori* structure of the modified WIHIC questionnaire comprising 24 items in actual form, with 8 items in each of the three scales (see Appendix B). The factor loadings obtained are in Table 4.2.

No WIHIC item had a factor loading of less than 0.40 on its *a priori* scale or more than 0.40 on either of the other two WIHIC scales. This demonstrates that this survey has a strong factor structure. The percentage of total variance extracted ranged from 7.25% to 30.63% for all the scales and the eigenvalue associated with each factor

ranged from 1.74 to 7.35. The total variance was 48.65% for the three scales of WIHIC (Involvement, Task Orientation, and Investigation). The percentage variance and the eigenvalue for each scale are shown at the bottom of Table 4.2. These data strongly support the factorial validity of the three-scale modified version of WIHIC actual form (Involvement, Task Orientation, and Investigation).

Table 4.2. Factor Loadings for a Modified Version of Actual Form of the WIHIC in California Middle Schools

Item Number	Factor Loadings		
	Involvement	Task Orientation	Investigation
1	0.68		
2	0.75		
3	0.50		
4	0.60		
5	0.56		
6	0.68		
7	0.51		
8	0.59		
9		0.67	
10		0.65	
11		0.57	
12		0.65	
13		0.69	
14		0.64	
15		0.75	
16		0.68	
17			0.61
18			0.62
19			0.74
20			0.65
21			0.74
22			0.73
23			0.79
24			0.66
% Variance	7.25	10.77	30.63
Eigenvalue	1.74	2.59	7.35

*N*=661

Factor loadings smaller than 0.40 have been omitted.

A principal components factor analysis with varimax rotation with Kaiser normalization also was performed for a sample of 661 middle-school mathematics students to confirm the *a priori* structure of the TOMRA comprising 20 items with

10 items in each of the two scales of Normality of Mathematicians and Enjoyment of Mathematics (see Appendix C). The factor loadings obtained are shown in Table 4.3. This analysis was performed to identify faulty items that could be removed in order to improve the internal consistency reliability and factorial validity of the two scales of TOMRA used in this study.

Table 4.3. Factor Loadings for TOMRA in California Middle Schools

Item Number	Factor Loadings	
	Normality of Mathematicians	Enjoyment of Mathematics
N7	0.54	
N11	0.79	
N15	0.55	
N19	0.71	
E2		0.61
E4		0.71
E6		0.57
E8		0.69
E10		0.67
E12		0.54
E14		0.69
E16		0.48
E18		0.63
E20		0.66
% Variance	22.58	9.50
Eigenvalue	4.52	1.90

*N*=661

Factor loadings smaller than 0.4 have been omitted.

E is the Enjoyment of Mathematics items.

N is the Normality of Mathematicians items.

Six items appeared to be problematic for the students. Items 1, 3, 5, 9, 13, and 17 were dropped because their factor loading was less than 0.40 on the *a priori* scale. Removal of Items 1, 3, 5, 9, 13, and 17 from Normality of Mathematicians scale enhanced the internal consistency reliability and factor structure of the instrument. All the 14 remaining items had a factor loading of at least 0.40 on their *a priori* scale or larger with the item's own scale and less than 0.40 with the other TOMRA scale. The percentage of variance extracted was 22.58% for Normality of Mathematicians and 9.50% for Enjoyment of Mathematics, making a total of 32.08%. The eigenvalue associated with the two factors are 1.90 to 4.52. The percentage variance and the

eigenvalue for each scale are shown at the bottom of Table 4.3. Overall the results in Table 4.3 support the factor structure of a two-scale, 14-item version of TOMRA.

A scree test can be useful in determining the total amount of variance explained by each component (French & Chess, 2002). The eigenvalues for successive factors can be displayed in a simple line plot with the principal components on the  $x$ -axis. According to Cattell (1966), if the principal components are plotted according to their sizes, as a diminishing series and the points are connected, a relatively sharp break appears where the true number of factors ended and the ‘detritus’, presumably due to error factors, appears. Cattell (1966) proposed that this scree plot can be used to determine graphically the optimal number of factors to retain. Using this criterion, a decision about which components to include is made by looking at the leveling off when the factors are mainly measuring random errors. Therefore, Cattell’s scree test involves finding the place where the smooth decrease of eigenvalues appears to level off to the right of the plot. Presumably, one finds only the ‘factorial scree’ to the right of this point. To ascertain the number of factors extracted using the Kaiser criterion in my study, Cattell’s scree test was applied to verify the number of factors retained. Figures 4.1 to 4.3 show the scree plot for the CLES, WIHIC, and TOMRA, respectively. These scree plots show a distinct break (inflection point) between the steep slope of the larger factors and the gradual trailing off of the rest of the factors. The principal components to be retained in my study are those to the left of the distinct break or inflection point.

Three components were extracted from the CLES and the scree plot depicting the point where the extraction leveled out is shown in Figure 4.1. The WIHIC had three components extracted. Three components were extracted from the two scales of the TOMRA questionnaire used for the study according to this criterion. This implies that the two scales of the TOMRA (Normality of Mathematicians and Enjoyment of Mathematics) did not measure just the two distinct variables, but more than two constructs in the attitude questionnaire. The scree plot could be a supporting criterion for extracting factor components for the CLES, WIHIC and TOMRA.

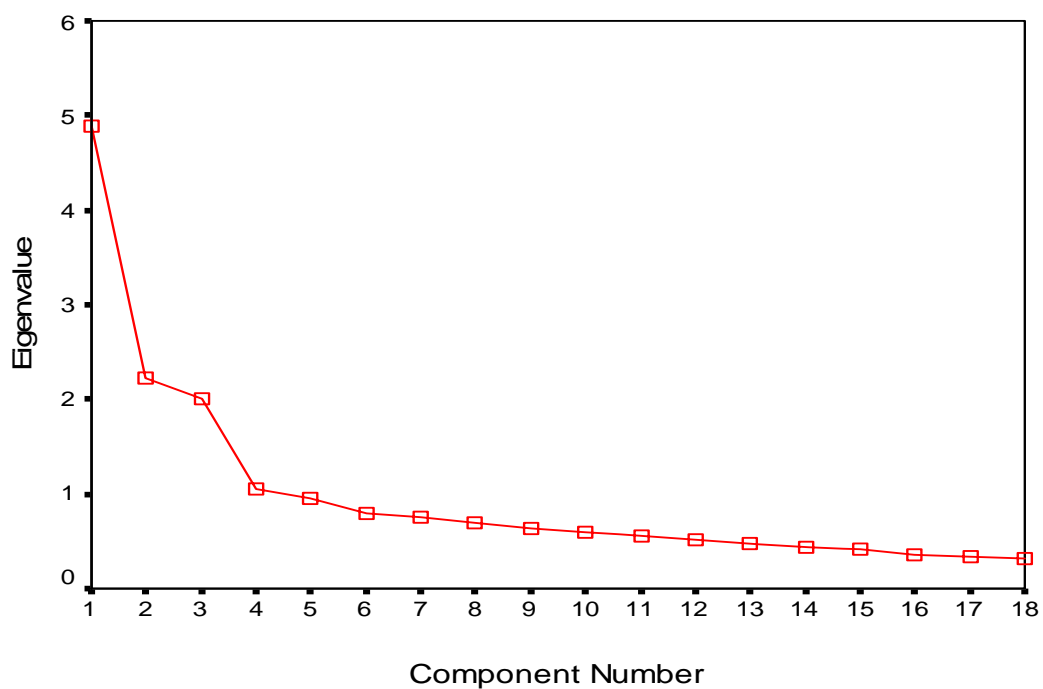


Figure 4.1. Three-Scale Scree Plot for CLES

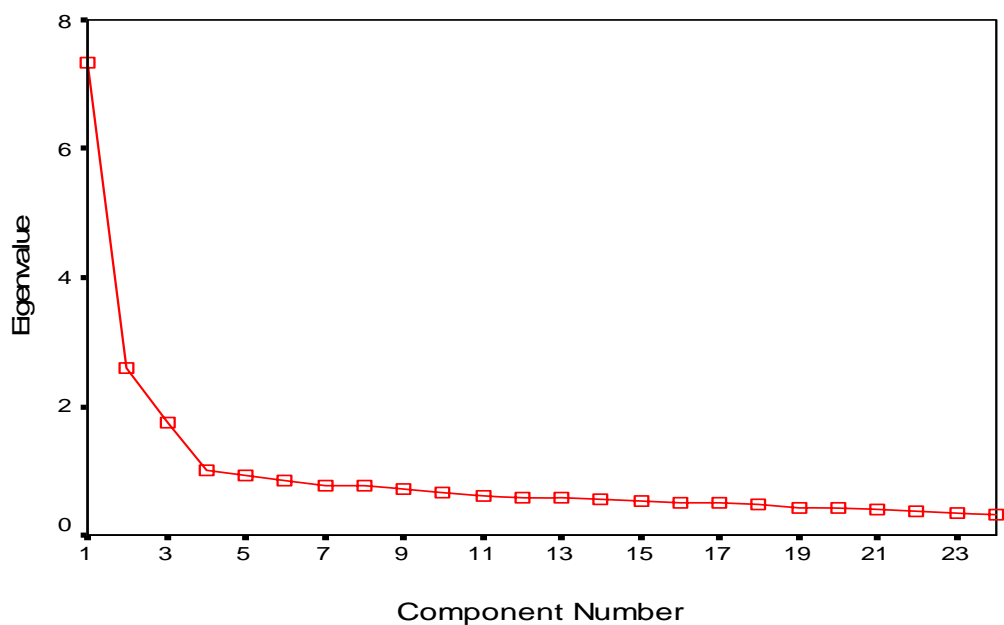


Figure 4.2. Three-Scale Scree Plot for WIHIC

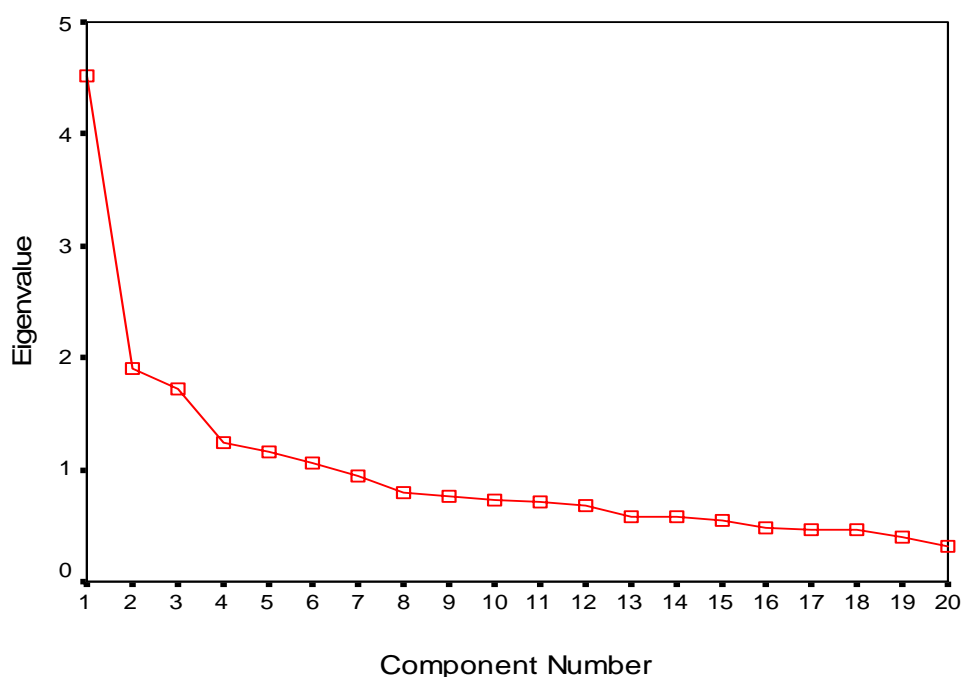


Figure 4.3. Two-Scale Scree Plot for TOMRA

#### 4.2.2 Internal Consistency and Discriminant Validity of CLES, WIHIC, and TOMRA

Internal consistency reliability analysis is commonly used to provide a measure of scale reliability and also to provide information about the relationships between individual items in the scale (i.e. whether each item within a scale is assessing a common construct). The data from the CLES, WIHIC, and TOMRA were subjected to scale internal consistency analysis to investigate the extent to which items in the same scale measure a common construct. The internal consistency for each scale was determined using the Cronbach's alpha coefficient for the sample of 661 students for the two classroom environment questionnaires (CLES and WIHIC) and the attitude scales (TOMRA) for two units of analysis (individual and class means).

Table 4.4 reports the internal consistency reliability (Cronbach alpha coefficient) for the CLES, WIHIC, and TOMRA. The results presented in Table 4.4 show that the alpha coefficients for different CLES scales were high, ranging from 0.71 to 0.84 using individual as the unit of analysis and from 0.83 to 0.93 for class means. The reliability of different WIHIC scales ranged from 0.81 to 0.88 using the individual as

the unit of analysis and from 0.90 to 0.94 for class means. The reliabilities for the attitude scales (Normality of Mathematicians and Enjoyment of Mathematics) were 0.64 to 0.82 for the individual as the unit of analysis and 0.86 to 0.89 for class means. The alpha reliability coefficient for every scale in Table 4.4 is well above the minimum value of 0.50 suggested for scale scores to be meaningful (Cronbach, 1951).

Table 4.4. Internal Consistency Reliability (Cronbach Alpha Coefficient), Discriminant Validity (Mean Correlation with Other Scales) for Two Units of Analysis and Ability to Differentiate between Classrooms (ANOVA) for CLES, WIHIC, and TOMRA

Scale	Unit of Analysis	Alpha Reliability	Mean Correlation with Other Scales	ANOVA Eta <sup>2</sup>
CLES				
Personal Relevance	Individual	0.71	0.29	0.06
	Class Mean	0.83	0.46	
Shared Control	Individual	0.84	0.32	0.07**
	Class Mean	0.93	0.54	
Students Negotiation	Individual	0.83	0.39	0.09**
	Class Mean	0.93	0.52	
WIHIC				
Involvement	Individual	0.81	0.42	0.06**
	Class Mean	0.90	0.57	
Task Orientation	Individual	0.84	0.37	0.08**
	Class Mean	0.94	0.56	
Investigation	Individual	0.88	0.38	0.06**
	Class Mean	0.94	0.45	
TOMRA				
Normality of Mathematicians	Individual	0.64	0.58	
	Class Mean	0.89	0.63	
Enjoyment of Mathematics	Individual	0.82	0.58	
	Class Mean	0.86	0.63	

\*\* $p < 0.01$

$N=661$  students in 22 classes in California.

The eta<sup>2</sup> statistics (ratio of 'between' to 'total' sums of squares) represents the proportion of variance explained by group membership.

The discriminant validity (extent to which a scale measures a distinct construct that is not assessed by the other scales) or scale independence, was determined for each questionnaire using the mean correlation of a scale with the other scales as a

convenient index. Discriminant validity values in Table 4.4 are quite high in some cases, suggesting that some scales overlap. The discriminant validity values for CLES ranged from 0.29 (Personal Relevance) to 0.39 (Student Negotiation) for the individual as the unit of analysis and from 0.46 (Personal Relevance) to 0.54 (Shared Control) for class means.

Similarly, the mean correlation of a scale with other scales of the WIHIC ranges from 0.37 (Task Orientation) to 0.42 (Involvement) for the individual as the unit of analysis and from 0.45 (Investigation) to 0.57 (Involvement) for class means. The correlation between the two TOMRA scales of Normality of Mathematicians and Enjoyment of Mathematics was 0.58 for the individual as the unit of analysis and 0.63 for class means. These data suggest that raw scores on the scales overlap, but not to the extent that the psychometric structure of the survey instrument is violated. Moreover, the factor analysis results reported in Tables 4.1 to 4.3 attest to the independence of factor scores. These results compare favorably with discriminant validity data for other well-established classroom environment instruments (see Fraser, 1998b) and other recent studies (see Dorman, 2003; Raaflaub et al., 2002; Sebela et al., 2004).

#### **4.2.3 Ability of CLES and WIHIC to Differentiate Between Classrooms**

A one-way analysis of variance (ANOVA) was calculated for each scale of the CLES and WIHIC to investigate its ability to differentiate between the perceptions of students in different classrooms. For each ANOVA, scores on a classroom environment scale were used as the dependent variable and class membership was the independent variable. Eta-squared ( $\eta^2$ ) is interpreted as the proportion of the total variability in the dependent variable (classroom environment scores) that is accounted for by variation in the independent variable (class membership). It is the ratio of the between-groups sum of squares to the total sum of squares.

The value of  $\eta^2$  in Table 4.4 for the sample of 661 students ranged from 0.06 to 0.09 for the scales of the CLES and WIHIC. The ANOVA results were statistically significant ( $p < 0.01$ ) for every scale except the CLES scale of Personal Relevance. This suggests that most scales of the CLES and WIHIC are able to differentiate



between the perceptions of students in different classes. These findings suggest that students perceive the learning environments of different mathematics classrooms differently.

#### **4.3 COMPARISON OF EXPERIMENTAL AND CONTROL GROUPS ON ACHIEVEMENT, CLASSROOM ENVIRONMENT AND ATTITUDES**

The data collected from the pretest and posttest administrations were statistically analyzed to answer the second research question:

Research Question # 2:

*Is an innovative teaching approach – involving the use of information technology, numerical methods (Cramer’s rule), and constructivist methods for the topic of systems of linear equations – effective in terms of promoting:*

- (a) a positive classroom environment*
- (b) student attitudes to mathematics*
- (c) student achievement and ability to identify and apply concepts?*

This section mainly reports pretest-posttest changes in classroom environment perceptions, students’ attitudes and students’ achievement for the subgroups of students who experienced the innovative methods and for a control group. The sample size for the experimental group was 61 students and for the comparison group (control) was 40 students. There are limitations associated with this small sample size in terms of the statistical power of analyses and the generalizability of results. The results of a comparison of students’ achievement for the experimental and control groups are presented in Table 4.5. Later, differences between pretest and posttest scores for CLES and WIHIC scales and one aspect of achievement are shown in Table 4.6 for the two groups. This section is organized according to the following topics:

- Comparison of experimental and control groups on achievement  
(Section 4.3.1)
- Changes in classroom environment perceptions, attitudes, and achievement for experimental and control groups (Section 4.3.2)
- Differential changes experienced by experimental and control groups in

classroom environment perceptions, attitudes and achievement (Section 4.3.3).

#### **4.3.1 Comparison of Experimental and Control Groups on Achievement**

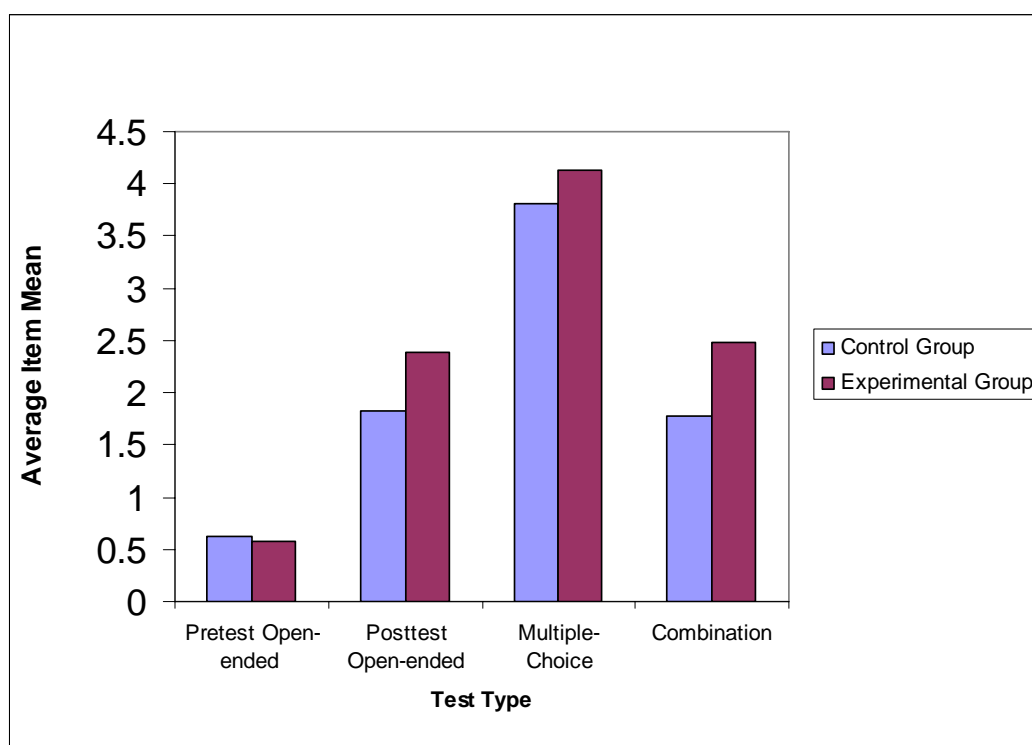
In this section, students' achievement for the experimental group and for the control group is compared. The sample consisted of 61 students in the experimental group and 40 students in the control group. The statistical power is limited in these data analyses due to the small sample size, and this is recognized as one of the limitations of this study. There were three types of achievement measures used in my study in order to evaluate the students' conceptual understanding and mastery of the systems of linear equations:

- open-ended questions
- multiple-choice questions
- combination questions involving constructive responses.

The open-ended questions are the only ones that were administered as an identical pretest and posttest. The multiple-choice questions and combination (constructive-response questions) were administered only once. Because open-ended items alone are inadequate to measure accurately the students' achievement, different test formats (Moschkovich, 1999) involving multiple-choice questions and constructive-response questions (for which students answer questions on real-life problem situations) were used to capture holistically the students' achievements for the two groups (control and experimental). The use of a variety of assessment strategies has been recommended by different researchers (eg. Treagust et al., 2003) in order to obtain a clearer evaluation of the students.

Figure 4.4 depicts graphically the mean score for the experimental and control groups for different achievement tests. Figure 4.4 shows that there is little difference in the pretest scores of the experimental and control groups on the open-ended questions. Although the pretest mean for the control group was slightly higher than the pretest mean for the experimental group, Figure 4.4 suggests that these groups of students had relatively similar levels of achievement before the study began. Also,

looking at the graphical profile as presented in Figure 4.4, it is observed that there are notable differences between the experimental and control groups for the other three achievement measures. In the posttest open-ended, multiple-choice, and combination achievement measures, the experimental group achieved better than the control group.



Pretest and Posttest are based on 5-point maximum score, Multiple-Choice is based on 9-point maximum score, and Combination method is based on 4-point rubric score.

Figure 4.4. Mean Score for the Experimental and Comparison Groups for Pretest and Posttest Achievement Test

Table 4.5 presents achievement test results for the control and experimental groups. In Table 4.5, students' achievement for the experimental and control groups are presented in terms of average item mean scores for different test types. Also, the average item standard deviation is shown for each achievement scale for the control and experimental groups. Table 4.5 also reports differences between the experimental and control groups in terms of both effect sizes (the difference in means expressed in standard deviation units) and the results of *t*-tests for the statistical significance of differences. The results in Table 4.5 show that differences between experimental and control groups are not statistically significant for the open-ended

questions at the time of pretesting (with a small effect size of 0.09 standard deviations), suggesting that groups were approximately equivalent on the pretest.

Table 4.5. Average Item Mean, Average Item Standard Deviation, and Differences Between Groups (Effect Size and *t*-test) for Different Assessment Measures for the Experimental and Control Groups

Achievement Type	Average Item Mean		Average Item Standard Deviation		Difference	
	Experimental	Control	Experimental	Control	Effect Size	<i>t</i>
Pre Open-Ended	0.65	0.72	0.80	0.92	0.09	-0.27
Post Open-Ended	2.49	2.03	1.24	1.43	0.34	2.12*
Multiple-Choice+	4.16	3.90	2.15	2.02	0.12	0.84
Combination++	2.53	1.86	0.97	1.22	0.61	2.68**

\*  $p < 0.05$

\*\*  $p < 0.01$

*N*=40 for the control group. *N*=61 for the experimental group.

+The maximum point is nine for multiple-choice questions.

++Based on four-point rubric scores.

The open-ended pretests and posttests are based on five-point maximum score.

However, Table 4.5 shows that the experimental group had significantly higher achievement scores than the control group for the open-ended questions on the posttest and for the combination questions. The effect sizes for these two measures are 0.34 and 0.61 standard deviations, respectively. These effect sizes suggest that the differences between the two groups were of moderate to medium magnitude. These results clearly indicate that the students who experienced the innovative strategy for learning systems of linear equations achieved better as recorded using two of the different assessment methods. There was no significant difference on the multiple-choice achievement measure between the experimental and the control groups (effect size of 0.12).

#### 4.3.2 Changes in Classroom Environment Perceptions, Attitudes and Achievement for Experimental and Control Groups

Differences between pretest and posttest scores on the three CLES scales, three WIHIC scales, two TOMRA scales, and the open-ended achievement measure were investigated using various methods. Only the open-ended achievement measure was administered as both a pretest and posttest.

First, descriptive statistics were generated for each scale, for each instructional group (experimental and control), and each testing session (pretest and posttest). The two descriptive statistics used were the average item mean and average item standard deviation, which are reported in Table 4.6.

Second, for each scale, the difference between pretest and posttest was calculated separately for the experimental and control groups using two methods. Whereas the effect size was calculated to provide a measure of the magnitude of pretest-posttest difference on each scale, a *t*-test for independent samples was used to estimate the statistical significance of the pretest-posttest difference for each of the nine scales (see Table 4.6).

Third, for each criterion (classroom environment, attitudes to mathematics, and mathematics achievement), an analysis of covariance (ANCOVA) was calculated with posttest scores on that criterion as the dependent variable, the corresponding pretest scores on that criterion as the covariate, and the group (experimental and control) as the independent variable. ANCOVA results are presented later in Section 4.3.3.

In order to estimate the magnitude of the differences between the pretest and posttest for the experimental and control groups for each CLES, WIHIC, TOMRA, and achievement scale, effect sizes were calculated in terms of the differences in means divided by the pooled standard deviation (Thompson, 1998a, 1998b). Table 4.6 shows that the effect size for different scales for the experimental group ranged from 0.02 to 1.99 standard deviations and for the control group ranged from 0.05 to 1.18 standard deviations for the control group. The effect sizes for the experimental group are somewhat larger for most scales than for the control group.

In terms of the statistical significance of the pretest-posttest difference on each scale, Table 4.6 reveals the following significant differences ( $p < 0.05$ ):

- the CLES scale of Shared Control (effect size of 0.31 standard deviations) for the experimental group but not for the control group

- the TOMRA scale of Normality of Mathematicians for both the control group (effect size of 1.18) and the experimental group (effect size of 1.99)
- the TOMRA scale of Enjoyment of Mathematics for the experimental group (effect size 0.61) but not for the control group (effect size of 0.21)
- the achievement measure for both the control group (effect size of 1.11) and the experimental group (effect size of 1.80 standard deviations).

Table 4.6. Average Item Mean, Average Item Standard Deviation and Difference between Pretest and Posttest (Effect Size and *t*-Test for Paired Samples) and ANCOVA Results for the CLES, WIHIC, and TOMRA Scales and Achievement for Control and Experimental Groups

Scale	Unit of Analysis	Average Item Mean		Ave. Item Standard Deviation		Pre-Post Difference		ANCOVA Results		
		Pre	Post	Pre	Post	Effect Size	<i>t</i>	Eta <sup>2</sup>	<i>F</i>	
CLES										
Personal Relevance	Control	3.05	3.01	0.71	0.64	0.06	-0.25	0.00	0.11	
	Experimental	3.05	3.06	0.65	0.68	0.02	0.03			
Shared Control	Control	2.10	2.14	0.83	0.88	0.05	0.25	0.03	2.34	
	Experimental	1.85	2.12	0.79	0.93	0.31	2.46*			
Student Negotiation	Control	3.06	3.23	1.02	0.92	0.18	0.95	0.02	2.18	
	Experimental	3.39	3.58	0.89	0.84	0.22	1.43			
WIHIC										
Involvement	Control	2.83	3.00	0.85	0.77	0.15	0.98	0.00	0.26	
	Experimental	2.97	3.09	0.85	0.83	0.14	1.00			
Task Orientation	Control	3.79	3.65	0.62	0.73	0.21	-1.17	0.05	5.52*	
	Experimental	3.91	4.01	0.71	0.68	0.14	0.86			
Investigation	Control	2.79	2.69	0.79	0.77	0.13	-0.67	0.00	0.27	
	Experimental	2.96	2.81	0.80	0.73	0.20	-1.23			
TOMRA										
Normality of Mathematicians	Control	2.77	3.70	0.98	0.60	1.18	5.07*	0.04	4.52*	
	Experimental	2.36	3.80	0.84	0.60	1.99	9.71**			
Enjoyment of Mathematics	Control	3.08	3.22	0.63	0.68	0.21	1.00	0.04	3.56*	
	Experimental	3.02	3.52	0.80	0.84	0.61	3.26**			
ACHIEVEMENT										
Open-ended	Control	0.72	2.03	0.92	1.43	1.11	6.32**	0.07	5.99*	
	Experimental	0.65	2.49	0.80	1.24	1.80	10.20**			

\*  $p < 0.05$

\*\*  $p < 0.01$

Experimental group  $N=61$

Control group  $N=40$

In general, the experimental group had larger effect sizes for the classroom environment scales, the attitudes scales, and open-ended achievement. The results as presented in Table 4.6 suggest that, relative to a control group, the group that experienced the innovative strategy for learning systems of linear equations generally experienced larger pretest-posttest changes than the control group in terms of their classroom environment perceptions and attitudes to mathematics and mathematics achievement.

#### **4.3.3 Differential Changes Experienced by Experimental and Control Groups in Classroom Environment Perceptions, Attitudes and Achievement**

Whereas Section 4.3.2 reported separately the pretest-posttest changes experienced by the experimental group and by the control group in terms of classroom environment perceptions, attitudes and achievement, this section reports the use of ANCOVA in examining whether the size of the pretest-posttest change in each criterion was bigger for the experimental group than for the control group.

An analysis of covariance (ANCOVA) was used to test the difference between the experimental and control groups on posttest scores on each CLES, WIHIC, TOMRA and achievement scale when pretest scores on the corresponding scale were held constant (i.e. used as a covariate). Differences in posttest scores for the classroom environment scales of CLES and WIHIC, the attitude scales of TOMRA, and achievement could be attributable not only to differences among the experimental and control groups, but also to the initial differences in pretest conditions. Unfortunately, these differences often threaten the internal validity of findings (Gay & Airasian, 2000). ANCOVA was used in order to control this source of variability. ANCOVA was also considered suitable for correcting the posttest achievement mean scores for existing pretest differences, as well as for reducing the amount of unexplained variance in the CLES, WIHIC, TOMRA, and achievement posttest scores, which Maxwell and Delaney (1990), Onwuegbuzie and Daniel (2003) and recently Veenman, Denessen, Van de Akker, and Van der Rijt (2005) claim could lead to an increase in the power of statistical tests.

Table 4.6 presents the ANCOVA results ( $\eta^2$  and  $F$  ratio) for each CLES, WIHIC, TOMRA, and achievement posttest scale when pretest was used as the covariate. The results of the ANCOVA indicate that there were statistically significant differences between the experimental and control groups on Task Orientation, Normality of Mathematicians, Enjoyment of Mathematics, and achievement ( $p < 0.05$ ). The magnitude of these effects ( $\eta^2$  statistic) was 0.05, 0.04, 0.04, and 0.07, respectively. Apart from Task Orientation, Normality of Mathematicians, Enjoyment of Mathematics, and achievement, the ANCOVA results for the other scales were not statistically significant, suggesting that the differences in mathematics classroom environment perceptions and attitudes were not different for the two instructional groups (experimental and control groups).

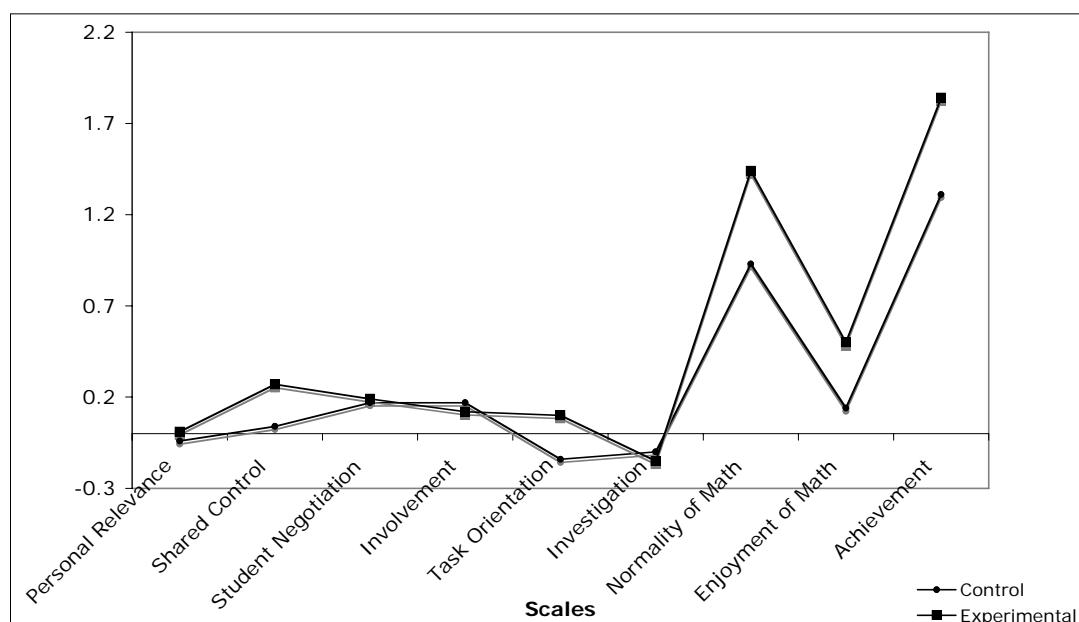


Figure 4.5. Pretest-Posttest Changes in Average Item Mean on the CLES, WIHIC, TOMRA and Achievement for the Experimental and Control Groups

In general, the analyses presented in Table 4.6 indicate that the innovative strategy for teaching and learning systems of linear equations could be beneficial in terms of pretest-posttest changes on Task Orientation, Normality of Mathematicians, Enjoyment of Mathematics and achievement. The ANCOVA results indicated that the magnitude of the pretest-posttest change experienced by the students in the experimental group was significantly different from the magnitude of the changes experienced by students in the control group on Task Orientation, Normality of



Mathematicians, Enjoyment of Mathematics, and achievement. These differential effects can be interpreted by examining the effect size and  $\eta^2$  in Table 4.6.

Figure 4.5 shows a graphical comparison of the control and experimental groups in terms of the magnitude of the pretest-posttest change in the average item mean for each environment scale (CLES and WIHIC), attitude scale (TOMRA), and achievement measure. In this figure, the magnitude of pretest-posttest changes for the experimental and control groups is consistent with the effect sizes reported in Table 4.6, which shows that the magnitudes of pretest-posttest changes generally are larger for the group using innovative strategies in teaching and learning systems of linear equations. Figure 4.5 shows that the pretest-posttest changes for Normality of Mathematicians, Enjoyment of Mathematics, and achievement for the experimental group tend to be larger than that for the control group. For Task Orientation, Table 4.5 shows that the experimental group experienced an improvement between pretest and posttest, whereas the control group experienced a decline.

#### **4.4 ASSOCIATIONS BETWEEN STUDENTS' ATTITUDES AND LEARNING ENVIRONMENT**

This section reports associations between students' attitudes to mathematics and their learning environment perceptions. The dependent variables were the Normality of Mathematicians and Enjoyment of Mathematics scales from TOMRA. For the present study involving a sample of 661 students in 22 classes, associations between students' attitudes and the learning environment were investigated using simple correlation and multiple regression analyses. All analyses were performed for two units of analysis (the individual and the class mean).

Whereas the simple correlation analysis provides information about bivariate associations between an attitude scale and an individual environment dimension, the multiple regression analysis provides a more parsimonious picture of the joint influence of a set of correlated environment scales on attitudes. The regression coefficients from the multiple regression analysis provide information about the magnitude of the relationship between an attitude scale and a particular environment scale when all of the other environment scales are mutually controlled.

Table 4.7. Simple Correlation and Multiple Regression Analyses for Associations Between Student Attitudes and Dimensions of CLES and WIHIC

Scale	Unit of Analysis	Attitude-Environment Association			
		Normality of Mathematicians		Enjoyment of Mathematics	
		<i>r</i>	$\beta$	<i>r</i>	$\beta$
CLES					
Personal Relevance	Individual	0.07	0.05	0.09*	0.00
	Class Mean	0.31	0.17	0.60**	0.00
Shared Control	Individual	-0.02	-0.08	0.13**	0.02
	Class Mean	0.15	0.13	0.51*	0.21
Student Negotiation	Individual	0.09*	0.06	0.23**	0.11*
	Class Mean	-0.12	-0.54	0.31	-0.13
WIHIC					
Involvement	Individual	0.09*	0.00	0.26**	0.09
	Class Mean	0.07	0.35	0.47*	0.58
Task Orientation	Individual	0.17**	0.16**	0.30**	0.21**
	Class Mean	0.17	0.19	0.43*	-0.19
Investigation	Individual	0.06	-0.01	0.21**	0.03
	Class Mean	-0.07	-0.13	0.27	-0.15
Multiple Correlation ( <i>R</i> )	Individual		0.19**		0.34**
	Class Mean		0.45		0.70

\*  $p < 0.5$ \*\* $p < 0.01$  $N=661$  students in 22 classes in California.

The results of the simple correlation and multiple regression analyses of attitude-environment associations are reported in Table 4.7 for two units of analysis. For the Normality of Mathematicians scale, Table 4.7 shows that the simple correlation is statistically significant for the CLES scale of Student Negotiation and the WIHIC scales of Involvement and Task Orientation with the students as the unit of analysis. Table 4.7 shows that the correlation between student Enjoyment of Mathematics and all three CLES scales of Personal Relevance, Shared Control, and Student Negotiation and all three WIHIC scales of Involvement, Task Orientation, and Investigation were statistically significant ( $p < 0.05$ ) for the individual as the unit of analysis. Although no simple correlation was statistically significant for Normality of Mathematicians with the class mean as the unit of analysis, the simple correlation for

class means for Enjoyment of Mathematics was statistically significant for the four scales of Personal Relevance, Shared Control, Involvement, and Task Orientation.

The multiple correlation ( $R$ ) reported in Table 4.7 for Normality of Mathematicians was 0.19 with the individual as the unit of analysis and 0.45 with the class means as the unit of analysis. The multiple correlation for Enjoyment of Mathematics for the three CLES and three WIHIC scales was 0.34 with the individual as the unit of analysis and 0.70 with the class means as the unit of analysis. The multiple correlation was statistically significant ( $p < 0.01$ ) for each attitude scale with the individual as the unit of analysis (but not with the class mean as the unit of analysis).

In order to ascertain which specific learning environment scales of the CLES and WIHIC account for most of the variance in attitudes scales when the other environment scales are mutually controlled, standardized regression weights ( $\beta$ ) were examined. As reported in Table 4.7, when using the Normality of Mathematicians as the dependent variable, the WIHIC scale of Task Orientation was significantly ( $p < 0.01$ ) and independently related to Normality of Mathematicians at the individual level of analysis. Also, the standardized regression coefficient show that the CLES scale of Student Negotiation and the WIHIC scale of Task Orientation were significantly ( $p < 0.05$ ), positively and independently related to Enjoyment of Mathematics with the individual as the unit of analysis.

Although the magnitudes of significant correlations in Table 4.7 are fairly small, these values clearly indicate that there is a moderate association between the learning environment and students' attitudes to mathematics for this group of middle-school mathematics students. Moreover, the sign of each significant correlation is positive. In particular, these results suggest that improved student attitudes are associated with more emphasis on all of the aspects of constructivism as assessed by the CLES, but especially Personal Relevance and Shared Control. In addition, the Task Orientation and Involvement scales of the WIHIC are linked with positive attitudes, suggesting that students enjoy their mathematics class more when they are involved and have clear task orientation. These results suggest that greater emphasis in the mathematics class on Task Orientation, Student Negotiation, and Involvement is linked with students enjoying their lessons more. The students as a whole seem to enjoy their

classes once they are able to be in control and involved in the class. These findings of outcome-environment associations replicate past research (Fraser, 1998b; Spinner & Fraser, 2005).

#### **4.5 GENDER DIFFERENCES IN CLASSROOM ENVIRONMENT PERCEPTIONS AND ATTITUDES TO MATHEMATICS**

Several researchers have suggested that students' perceptions of their classroom environment are related to the gender of the student (e.g. Henderson, Fisher & Fraser, 1995; Wong & Fraser, 1994). Gender differences in terms of classroom environment perceptions and attitudes to mathematics were investigated in my study in order to ascertain which gender group perceived their mathematics classes more positively. Descriptive statistics and MANOVA were performed for the sample of 661 students in order to determine gender differences in learning environment perceptions and attitudes to mathematics classes and achievement. The differences are presented in Table 4.8 and Figure 4.6.

A MANOVA was performed with gender as the dependent variable and the set of eight CLES, WIHIC, and TOMRA scales as well as achievement as the independent variables. Because the multivariate test using Wilks' lambda criterion suggested the existence of statistically significant gender differences for the set of dependent variables as a whole, the univariate ANOVA was interpreted separately for each of the eight environment and attitudes scales.

Table 4.8 includes for each scale the average item mean and average item standard deviation, as well as the effect size and ANOVA result for gender differences. The average item mean is calculated by dividing the scale mean by the number of items in the scale. It provides a useful basis for comparison between scales containing differing numbers of items. The average perceptions of each gender group is reported in Table 4.8 and shown graphically in Figure 4.6. The average item standard deviation of each of the eight scales is included in Table 4.8 as a measure of the extent to which the scores deviate from their mean for each scale.

Table 4.8. Average Item Mean, Average Item Standard Deviation, and Gender Difference (Effect Size and MANOVA Results) for Each CLES, WIHIC, and TOMRA Scale and Achievement

Scale	Average Item Mean		Average Item Standard Deviation		Gender Difference Effect Size	
	Females	Males	Females	Males	Size	<i>F</i>
<b>CLES</b>						
Personal Relevance	3.10	3.14	0.71	0.72	0.06	0.38
Shared Control	2.03	2.06	0.85	0.91	0.04	0.15
Student Negotiation	3.32	3.09	0.95	0.99	0.24	6.61*
<b>WIHIC</b>						
Involvement	3.06	2.92	0.81	0.85	0.16	2.96
Task Orientation	4.02	3.88	0.72	0.83	0.18	3.86*
Investigation	2.85	2.88	0.92	0.84	0.04	0.15
<b>TOMRA</b>						
Normality of Mathematicians	3.36	3.35	0.73	0.74	0.02	0.04
Enjoyment of Mathematics	3.38	3.37	0.88	0.92	0.01	0.02
<b>ACHIEVEMENT</b>						
Open-ended	2.47	2.53	1.31	1.16	0.07	1.84

\* $p < 0.05$

$N = 661$

The effect size (i.e. the difference between means divided by the pooled standard deviation) provides information about the magnitude of a difference (Raaflaub et al., 2002; Sebela et al., 2004; Spinner & Fraser, 2005). Effect sizes were calculated as recommended by Thompson (1998a, 1998b). They are included in Table 4.8.

As shown in Table 4.8, there are statistically significant ( $p < 0.05$ ) gender differences occurring only for the CLES scale of Student Negotiation and the WIHIC scale of Task Orientation. The effect sizes for the two significant gender differences are relatively small with values of 0.24 standard deviations for Student Negotiation and 0.18 standard deviations for Task Orientation (Table 4.8). The female average item mean was higher than the male average item mean for these two scales, with female scoring 3.32 for Student Negotiation and 4.02 for the Task Orientation, while the male students scored 3.09 and 3.88, respectively. There is no statistically significant difference between the genders on achievement, which is consistent with recent

studies (e.g. Kijkosol & Fisher, 2005). However the female average item mean was higher than the male average item mean on achievement, with females scoring 2.62 and the male students scoring 2.53. Female students had more favorable classroom environment perceptions but lower achievement scores than males based on these findings.

Figure 4.6 graphically illustrates these findings showing that overall there are small differences between the male and female students in their attitudes to mathematics, classroom environment perceptions, and achievement. As depicted in Figure 4.6, there are some moderate gender differences in classroom environment perceptions in favor of females, especially on the CLES Student Negotiation scale and the WIHIC Task Orientation scale.

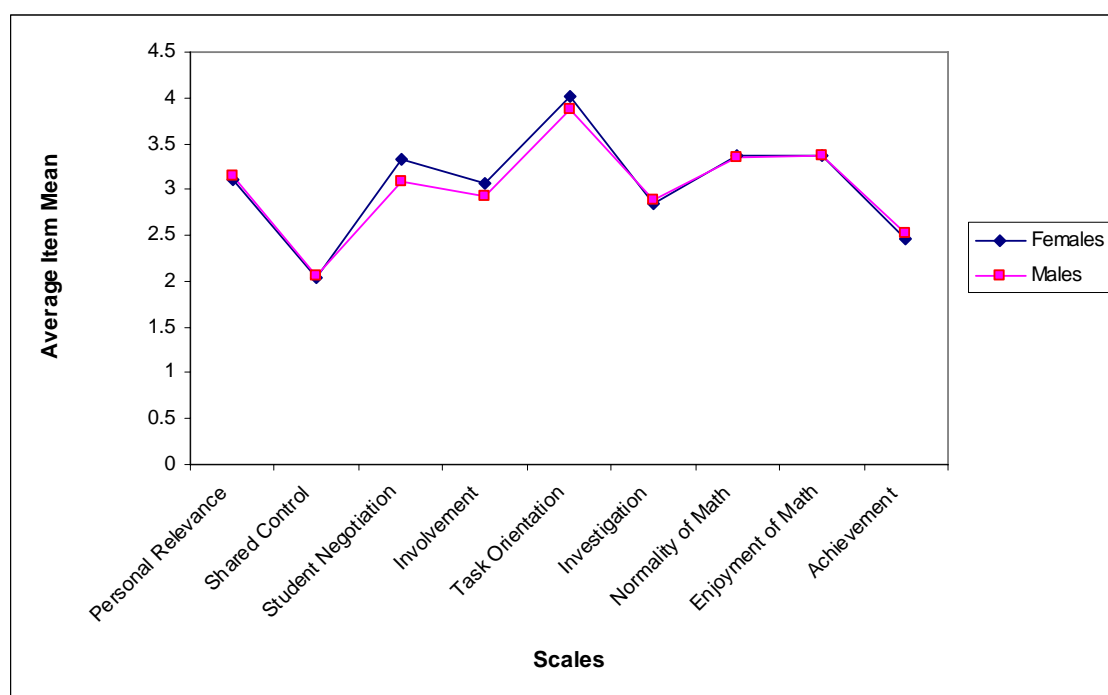


Figure 4.6. Gender Differences for Classroom Environment Perceptions, Attitudes to Mathematics and Achievement

These results suggest that the female students perceive their mathematics classroom somewhat more positively than the male students for this group of California mathematics students. These findings replicate past research (Chen et al., 2002; Henderson et al., 1995; Margianti et al., 2002; Raaflaub & Fraser, 2002; Riah & Fraser, 1998).

#### **4.6 LIMITATIONS OF MY STUDY**

While my study has some strengths, it is very important also to point out its limitations. First, the statistical power is limited in some data analyses due to the small sample size. In particular, a subsample of only 101 students comprised the control and the experimental groups. In addition to the sample size which might have limited the statistical power for most of the analysis, the representativeness of the sample could limit the generalizability of the findings and should be recognized as a limitation to be kept in mind when interpreting the findings from this study. Moreover, because of the smallness of the sample size for the comparison of experimental and control groups, it was not possible to use MANOVA and MANCOVA. My use of multiple *t*-tests and multiple ANCOVAs could have given rise to Type I errors.

Second, the teacher being the researcher in this study could have given rise to bias and errors in interpreting some of the results. The researcher might not have been an impartial observer. The form of researcher bias that is commonly more prevalent at the data analysis stage is the halo effect which, according to Onwuegbuzie and Daniel (2003), occurs when a researcher is scoring open-ended responses, or the like, and allows her or his prior knowledge of or experience with the participants to influence the scores given. It was not always possible to have other teachers of mathematics observe the experimental group classes during the study. If this were possible, their feedback and critique would have been valuable in enhancing the quality of this study. Seeing what the researcher wants to see in the data could have obscured the sound judgment of the researcher.

Third, the representativeness of the sample could be another limiting factor in that, when compared to the general eighth-grade population in California, my sample could be considered neither a sizeable fraction of the population nor representative of the full range of schools and students. This limits the generalizability of the findings.

## 4.7 SUMMARY OF THE ANALYSES AND RESULTS

Chapter 4 presented the results and analyses of the study based on quantitative data-gathering methods. In order to answer the research questions, a sample of 661 eighth-grade middle-school students in 22 classrooms from California and a subsample of 101 students participate in the study. Chapter 4 focused on reporting the results of the:

- Validity and reliability of the CLES, WIHIC and TOMRA (Section 4.2)
- Comparison of experimental and control groups' achievement, classroom environment and attitudes (Section 4.3)
- Associations between students' attitudes and learning environment (Section 4.4)
- Gender differences in classroom environment perceptions and attitudes to mathematics (Section 4.5)
- Limitations of my study (Section 4.6).

In order to answer the first research question, involving whether scales assessing classroom environments and attitudes to mathematics are valid when used with middle-school students in California, data were collected from the administration of the CLES, modified WIHIC, and TOMRA scales to a sample of 661 eighth-grade students. The data gathered were statistically analyzed to provide evidence about the factor structure, internal consistency reliability, discriminant validity, and ability of the CLES and WIHIC to differentiate between classrooms using one-way ANOVA. The findings are summarized below:

*Finding 1: The CLES, WIHIC, and TOMRA displayed satisfactory factorial validity. The total percentage of variance extracted was 50.73% for the three CLES scales, 48.65% for the three WIHIC scales, and 32.08% for the two TOMRA scales.*

*Finding 2: The CLES, WIHIC, and TOMRA demonstrated satisfactory internal consistency reliability for two units of analysis (individual and class mean).*



*Finding 3: Discriminant validity results (using the mean correlation of a scale with other scales as a convenient index) for two units of analysis show that raw scores on scales overlap, but not to the extent that the psychometric structure of the instruments is violated. Moreover, the factor analysis results support the independence of factor scores on the scales in each of the CLES, WIHIC and TOMRA.*

*Finding 4: The CLES and WIHIC scales can differentiate significantly between the perceptions of students in different classrooms.*

In order to answer the second research question, changes in pretest and posttest scores were analyzed separately for 61 students in the experimental group and 40 students in the control group using effect size and *t*-test paired samples. These analyses were conducted to investigate if an innovative teaching approach – involving the use of numerical methods (Cramer’s rule) and constructivist methods for the topic of systems of linear equations – effective in terms of promoting (a) a positive classroom environment, (b) student attitudes to mathematics, and (c) student achievement and student ability to identify and apply concepts.

*Finding 5: Statistically significant pretest-posttest differences between the experimental and control groups were evident for:*

- *the CLES scale of Shared Control for the experimental group*
- *the TOMRA scale of Normality of Mathematicians for both the experimental and control groups*
- *the TOMRA scale of Enjoyment of Mathematics for the experimental group*
- *open-ended achievement for both the experimental and control groups.*

*Finding 6: In general, relative to the control group, the experimental group showed somewhat larger changes for the classroom environment scales and attitudes scales, as well as for open-ended achievement.*

ANCOVA was also conducted to determine the differential changes experienced by the experimental and control groups in classroom environment perceptions, attitudes,

and achievement when the pretest scores on the corresponding scale is held constant (covariate).

*Finding 7: There were statistically significant differential pretest-posttest changes for the experimental and control groups for the WIHIC scale of Task Orientation, the TOMRA scales of Normality of Mathematicians and Enjoyment of Mathematics and achievement. The experimental group experienced larger pretest-posttest changes.*

The findings for the third research question involving associations between the classroom environment and student attitudes to mathematics are summarized below:

*Finding 8: A positive and statistically significant correlation exists between Normality of Mathematicians and the CLES scale of Student Negotiation and the WIHIC scales of Involvement and Task Orientation with individual as a unit of analysis (but not at the student level).*

*Finding 9: There is a positive and statistically significant correlation between Enjoyment of Mathematics and all three CLES and three WIHIC scales with the student as a unit of analysis, for the four scales of Personal Relevance, Shared Control, Involvement, and Task Orientation with the class mean as the unit of analysis..*

*Finding 10: The multiple correlation between the group of three CLES and three WIHIC scales and each of the two TOMRA scales is statistically significant for the individual as a unit of analysis.*

*Finding 11: Improved student attitudes are associated with more emphasis on all of the aspects of constructivism, especially Personal Relevance and Shared Control, as well as the WIHIC scales of Task Orientation and Involvement.*

Finally, the results for the fourth research question, which addressed gender differences in perceptions of classroom environments, attitudes to mathematics and achievement, are summarized below:

*Finding 12: A statistically significant but small difference was found between the genders for Student Negotiation and Task Orientation. Female students perceived their mathematics classroom somewhat more positively than did the male students for this group of Californian mathematics students. There was no statistically significant difference between the genders for achievement or students' attitudes to mathematics.*

The limitations of my study were discussed briefly in this chapter and are revisited in more detail in Chapter 6.

In this chapter, findings from quantitative data-gathering methods are reported in terms of classroom environment, students' attitudes, and students' conceptual development. In the next chapter, the perceptions of the students who were involved in the experimental group and control group are explored more closely by analyzing qualitative information obtained using audiotaped interviews, students' journals, and case studies involving analysis of the students' work.

## Chapter 5

### QUALITATIVE DATA ANALYSIS

#### 5.1 OVERVIEW

In this chapter, the qualitative data analyses and results are presented in five main sections. The first section introduces the chapter. Section 5.2 discusses the audiotaped interviews obtained from both the students who experienced the innovative strategy and the control group. In the third section, the information gathered from the students' learning journals is presented. Section 5.4 presents analyses of students' work samples. The last section in this chapter summarizes the findings involving qualitative data.

In this chapter, I report the qualitative data collected through ethnographic techniques involving audiotapes of interviews with students, analysis of students' journals and reflective writing, and analysis of students' work samples. Twelve students, comprising eight from the experimental group and four from the control group, were interviewed privately. Students' reflective journals were collected and analyzed to get a better insight into the students' thoughts about the topic of systems of linear equations and what they thought regarding the methods of solving problems involving systems of linear equations. In addition, students' work samples were randomly selected and analyzed in order to examine students' errors and misconceptions as they learned the topic of systems of linear equations.

Tobin, Kahle, and Fraser (1990) reported that the use of qualitative methods in learning environment research provided them a more in-depth understanding of learning environments. Also, Spinner and Fraser (2005) used qualitative methods to provide insights into students' perceptions. Analysis of students' work samples as a qualitative information-gathering method has been reported for the topics of linear equations (Pirie & Martin, 1997), natural number domain to the integers (Gallardo, 2002), algebra (Clement, 1980, 1982; Matz, 1982), the  $x$ -intercept (Moschkovich, 1999), graphs (Bell & Janvier, 1981), and the concept of conservation of area (Kordaki, 2003). The aim of this mixed-methods research is to draw from the

strengths, while minimizing the weaknesses, of both quantitative and qualitative research methods in a single research study (Johnson & Onwuegbuzie, 2004).

This chapter is organized according to the following topics:

- Overview (Section 5.1)
- Audiotaped interviews (Section 5.2)
- Students' learning journals (Section 5.3)
- Analysis of the students' work (Section 5.4)
- Summary of qualitative data findings (Section 5.5).

## **5.2 AUDIOTAPED INTERVIEWS**

Interviews were conducted with eight students selected from the experimental group and four students selected from the control group. Interviews have long been recognized as an effective way to collect information about a student's mathematical concepts and skills, and as a way to gain qualitative data about an individual (Ashlock, 1998), as well as a way to gather information about student attitudes and learning environment perceptions. Although the student interviews were limited in length and depth in that the study mainly involved quantitative data-collection methods, extensive qualitative information was obtained through the students' journals and reflections in order to complement the interview data. The interview script (Figure 5.1) – with some questions adapted from Spinner and Fraser (2005) – was used to interview these students. Each group of students was chosen randomly.

The qualitative data obtained through audiotaped interviews provided a deeper understanding of students' perceptions of their classroom environment, their attitudes to mathematics, and their conceptual development. Burns (1997) employed research methods that drew on ethnographic techniques, such as interviews, survey instruments, and the collection of video and auditory records, in an investigation of students' and teachers' use of technology in specific classroom environments. Duit, Treagust and Mansfield (1996), on the other hand, explained that, in naturalistic

settings, teaching and learning events are often audiotaped and verbatim transcripts are made of the lesson or activity. Tobin and Fraser (1998) explained that, when a

This interview aims to help your teacher to understand your thoughts about your classroom. I will ask you 16 questions and I want your responses to be truthful so that I can make changes to improve your learning of algebraic concepts and your learning environment.

- |     |   |
|-----|---|
| 1.0 | Students' roles   |
| 1.1 | In this class, why do you have to learn algebra?  |
| 1.2 | In this class, how do you help the teacher to plan your algebra activities?   |
| 2.0 | Teacher's roles   |
| 2.1 | What would you recommend the teacher do to be sure that each student receives individualized attention (differentiated instruction)?                                  |
| 2.2 | What would you prefer the teacher to do if students are having trouble with class activities?   |
| 3.0 | Classroom environment   |
| 3.1 | In this class, how does communicating with other students help to enhance the learning environment?   |
| 3.2 | In this class, why do you need to express your opinions?  |
| 4.0 | Students' attitudes   |
| 4.1 | Explain the method you like the most for solving systems of linear equations and why do you prefer that method?   |
| 4.2 | Do you like to use new mathematics methods which you have not used before? Why?   |
| 5.0 | Conceptual development  |
| 5.1 | Why is finding out new algebra concepts important?  |
| 5.2 | Tell me some ways that help you to understand algebra concepts?   |
| 6.0 | System of linear equations concepts   |
| 6.1 | How have you enjoyed the topic of systems of linear equations?  |
| 6.2 | Did Cramer's rule make solving systems of linear equations easier? Why?   |
| 7.0 | Use of technology   |
| 7.1 | How does the use of the computer help you to understand some algebra concepts?  |
| 7.2 | Did you learn and understand better the concept of systems of linear equations when you had the opportunity to use U-Math software?                                   |
| 8.0 | Conclusion  |
| 8.1 | Has your interest in algebra somewhat improved through the use of simple methods, such as combination and Cramer's rule, for solving SLE and through using computers? |
| 8.2 | In conclusion, what have been the most important experiences in helping you to like or dislike your algebra class?  |

Figure 5.1. Interview Script

study using quantitative methods has been completed, its main findings can be contextualized with thick descriptions consisting of observations and verbal accounts

from participants. Subsequently, qualitative techniques can be used to obtain insights into particular educational, social, and familial processes and practices that existed within a specific location (Connolly, 1998).

The transcribed interview script revealed the students' thinking about their roles in the class as an aspect of constructivism, as well as their thoughts about their classroom environment, attitudes, and conceptual development. The technique used in interpreting the interviews is similar to the approach suggested by Erickson (1998). This is presented according to the following subsections:

- Classroom environment (Section 5.2.1)
- Students' roles (Section 5.2.2)
- Students' attitudes (Section 5.2.3)
- Conceptual development (Section 5.2.4).

### **5.2.1 Classroom Environment**

In response to the classroom environment questions, students responded that they believe that they should be helping each other in class and also that the teacher should allot a certain amount of time for each group of students so that they always are engaged. In addition, the teacher should ask the students questions to check for understanding and help those students who need most help. When asked the question "In this class, how does communicating with other students help to enhance the learning environment?", different and varied answers were obtained from these students. The majority of the students think that helping each other is important in fostering a good classroom environment. A female student said that communicating with other students helps to enhance the learning "because you could help one another...and, if you need help in one problem, I help you and, if I need help with a problem, you help me".

In responding to the question "In this class, why do you need to express your opinion?", a student said that, "...when you do a problem, there are almost two or three ways to do it. So, sometimes you might have someone who could say "I have an easier way, you did it wrong why, you probably missed a step." Another student

responded by saying that “I think that we should express our opinion because it is important to learning”. Some of the students’ answered the WIHIC Involvement scale Item 1 and 2 during the interviews. Item 1 states “I discuss ideas in class” while Item 2 states “I give my opinions during class discussions”. A student suggested that, if you do not express your opinion, the teacher might not know that you need help and could continue to go on with the lesson. Also, they thought that they need to express their opinion in the class so that everyone can understand the same thing that you understood, everyone one can comprehend, and no-one would miss out. Similarly, the students thought that they need to express their opinion so that people could know what they are thinking regarding a mathematics problem. These perceptions of the students as regards to their involvement in the class corroborate the findings reported in Chapter 4.

### **5.2.2 Students’ Roles**

When students were asked “In this class, why do you have to learn algebra?”, most of the students interviewed said that they need to learn algebra because of its application in real life as well as its use in the future. They believe that they need to learn more so that they are prepared for high school. Some of the students suggested that, for almost any job that pays good money, “you have to use algebra”. In helping the teacher to plan their algebra activity as part of their role in the class, these students generally agreed that they should tell the teacher what help they need. One student said: “I tell the teacher what help I need and what is better for me... If I need something, I will ask him because... I want to make sure that I write it down.” In addition, students said that their role should include supporting the teacher’s ideas and thinking about how what they are learning could help them in life, as well as helping the teacher to select the activity on which to work in the class.

### **5.2.3 Students’ Attitudes**

In response to the students’ attitude interview questions, the students explained that the new topic had empowered them with many choices for solving a problem, especially problems involving systems of linear equations for which they can use Cramer’s rule not only because it is easy, but also because it is fun. Their attitudes



have improved in that they want to learn more and some of the students even want to be mathematics teachers. One student in particular explained that, at the beginning of the school year, he had no idea about what was going on in class. But, then, he decided that, if his grades are bad, that is a poor reflection on him. Therefore, he improved and now understands every lesson, especially the lesson on systems of linear equations. These students said that they like to use new mathematics methods especially Cramer's rule because they learned a different way to solve a problem so that they can pick the method that they like the most. A student said that he likes to use new mathematics methods because "I want to learn different mathematics steps. I don't just want to learn the same methods over and over again." Learning new mathematics methods will help them to be adequately prepared for high school. "It's great to learn different things and not just to be thinking about one thing so that you can have a choice between what to do in another situation", another student responded. In general, students think that learning new mathematics methods such as Cramer's rule will make life easier for them when solving systems of linear equations.

The interviews supplemented the information obtained through administering an attitude scale (TOMRA) to monitor student progress towards achieving attitude aims (Fraser, 1981a). That is, TOMRA was employed to obtain information about the mathematics-related attitudes of individual students and the whole class as they learned systems of linear equations using Cramer's rule. In Sections 4.5 and 4.6, a comparison of pretest and posttest attitude scores, as well as a comparison of the attitudes of experimental and control groups, provided some insights into changes occurring in student attitudes. Some of these changes can be better understood through what students said when they were interviewed. One student said that Cramer's rule helped her to improve her attitude towards mathematics. "Cramer's rule ... is easier and goes step by step and, if you get messed up..., you'll know right where you messed up and you can go back to that part without really having to do the whole thing." Another student said that Cramer's rule helped him to learn linear systems and this positively affected his attitudes towards mathematics. A female student said that "Cramer's rule ... is like fun while learning. I like Cramer's rule because you don't have to deal with many variables...because it's all numbers."

#### **5.2.4 Conceptual Development**

The use of innovative strategies in learning systems of linear equations was shown to be effective using quantitative data (Section 4.5). Therefore, findings from qualitative data gathering involving student interviews helped to provide further understanding regarding how students' conceptual understanding is supported by the use of an innovative strategy. It is the belief of some of the students interviewed that, if they want to be mathematics teachers at any level – for example – they need to learn many mathematics concepts so that they can teach their students. These students believe that their conceptual development was enhanced by asking the teacher questions, in order to get an easier way of solving mathematics problems.

Some of the students interviewed explained that taking notes helped them to understand algebra better. Their feeling is consistent with the Cornell note-taking strategy (Pauk, 1997). According to Pauk, the primary goal of note-taking is to provide you with a written record of what you've heard. The notes you jot down can become a handwritten textbook. These students think that writing down notes, like a memo, will help them to think about the problem that will aid them in mastering the concept. A female student answered the question about "...some ways that help you to understand algebra concepts" by saying "by taking notes and doing homework". Also, in answering the question "Why is finding out new algebra concepts important?", some students explained that it helps them to find ways to solve problems better in case they need them. In addition, they think that when they move on to a different grade, "you already know it". Consequently, they said, "...you can learn about many other things and not just one". They continued to say that "you need them in the future, when you are older". Also, it is important because "I need that in life to deal with the rest of the mathematics that I have to do in life and in college". In answering the question on "Tell me some ways that help you to understand algebra concepts?", a student said "studying more, paying more attention in class..." Also, some students think that the teacher should explain the problem more so that every student will understand it. In particular, a student suggested that "...You could say it in a song or you could repeat it over and over...you could see it everyday and every moment of the time..." Some students think that reviewing and

testing helped them to develop their conceptual understanding. When he was asked the question on “some ways that help you to understand algebra concepts” a student replied “doing work, activities, and maybe some computer homework...” Some of these students recognize how important the use of technology could be in learning, and how the use of computers could aid in their learning process (Masalski, 2005).

In general, the use of Cramer’s rule as a method in teaching and learning systems of linear equations helped some of these students who were interviewed to improve conceptually. Perhaps this was possible because the students in the experimental group had the opportunity to work collaboratively on an approach for solving SLE that they have not learned before which is also not in their algebra textbook. These findings gave insight into the findings from quantitative data regarding the students’ conceptual development when using the innovative strategies. This is consistent with Spinner and Fraser’s (2005) findings during the use of the *Class Banking System* for promoting conceptual development. These findings not only revealed students’ thoughts, attitudes and beliefs, but they also revealed to me as a teacher-researcher the ways in which I could change my beliefs, practice, and perceptions (Elliott, 1978; Stenhouse, 1975) in order to help my students better, especially my African American students. I might reach this group better by teaching them some algebra concepts as a song or as a rhyme. Subsequently, the use of a computer to complete homework was considered by these students interviewed to be effective in their conceptual development.

Several researchers have shown that teachers engaging in self-evaluation procedures should employ various feedback techniques (Bodine, 1973) and that teachers becoming more collaborative, collegial, and expert in their content knowledge leads to professional confidence and teacher efficacy (Garin, 2003; Hubbard & Power, 1999; Little, 1984). The rich working knowledge of the research setting (Spinner & Fraser, 2005) and close familiarity with the teaching-learning situation that I had as a teacher-researcher gave me advantages. Therefore, interviewing this group of randomly-selected students gave me additional insight into the ways in which my students learn in order to teach them more effectively.

On the other hand, a few of these students reported that Cramer’s rule as a method for learning systems of linear equations is long, complex, or confusing. One student

said that it takes up too many pages of her paper to complete one problem. Nonetheless, by looking at the advantages of learning systems of linear equations by Cramer's rule, its disadvantages as reported by these few students are few in the bigger picture.

### **5.3 STUDENTS' LEARNING JOURNALS**

In this section, a few learning journals including concept maps (see Appendix G) collected randomly from different students in the experimental group are discussed in order to gain insight into students' thinking as they learned systems of linear equations. Concept map was used also as an 'advance organizer' for learning in my classroom and as part of journal documentation of the topics they learned. Writing assignments are assessments that are produced by students, rather than teachers (Bush & Leinwand, 2000). They challenge students to think about their mathematical strengths and weaknesses, their attitudes, or their beliefs. They often take the form of student journals, students' reflections on their thinking and methods of working, and student-completed inventories (Bush & Leinwand, 2000). Also, Treagust et al. (2003, p. 37) claim that:

... use of individual writing tasks provides opportunities for each student to expand his or her personal ideas and reasoning, and reconcile them with accepted... concepts and processes. Individual writing tasks can capture students' understanding so that the teacher can assess their progress.

The students in the experimental group were given the writing prompt below:

In half a page, explain which method you prefer to use in solving a system of linear equations and why?

Responses indicate that students think that Cramer's rule is simple, easy to understand or learn, and interesting to learn. Some students think that the method helps them to identify mistakes and errors and is less complicated and less confusing. These students also said that using Cramer's rule makes solving problems faster, and involves a simple sequence. In general, students wrote that the steps involved in this method are more organized for easy comprehension, assimilation, retention and mastery. A step-by-step approach and the use of simple arithmetic, such as

multiplication and division, are involved. “At last, mathematics is better and more fun”, they said.

The students’ responses, as captured in their learning journals and as shown below, gave credence to this view. The approach used in discussing the students’ journals is similar to that of Gallardo (2002). Gallardo reported his interviews with quotations derived from students’ responses followed by an explanation of his findings based on what the students said. Most students believed that Cramer’s rule is a lot easier than the other methods under review (substitution, combination, and graphing methods). Max wrote in his journal that the other methods are confusing and that he learned Cramer’s rule faster than the other methods for learning systems of linear equations. He wrote: “This is the one I am going to use – Cramer’s rule – to solve problems. This is easier to solve.”

Another student, Steven, wrote:

I prefer Cramer’s rule. It is much easier to use. With substitution, I just don’t want to deal with fractions and with Cramer’s rule, you do not need to tamper with anything. You only use the basics of multiplication and division. That’s it. Isn’t that convenient? Graphing just isn’t my thing. I don’t have an endless supply of graph paper... but making the graph is annoying and time-consuming. Cramer’s rule is like the ‘in-n-out’ of mathematics.

Another student, Antonio, wrote that the method which he likes to use most is Cramer’s rule:

Cramer’s rule was easier for me to learn than graphing, substitution or combination. The reason that I like to use Cramer’s rule is that it is easy and it involves multiplication which I like to use. Also, Cramer’s rule takes less time to do. However, it takes up more space on the paper because it uses more numbers but they are easy.

Also, Mauricio wrote: “In mathematics system of linear equations, I like to use the Cramer’s rule method... because it easier.” For ease of understanding, the student, La Quisha, said:

Cramer’s Rule was easier than the rest. It’s better to understand and shorter to complete. All you have to do is take the coefficients from the equation and put them into the matrix. Then you have to find the determinant. You find  $D$  using the matrix and also use the matrix to find  $D_x$  and  $D_y$ . Then you divide  $D_x$  and  $D_y$  by  $D$  to get  $x$  and  $y$ . I would choose Cramer’s rule over any other method.

Most of these students who experienced using Cramer’s rule learned it faster and more easily, and learned to think that it is the best method. Britney wrote that:

...the method that I like the best is Cramer's rule. It is the easiest and best way. You just have to set it up, then solve and divide. Basically you just have to put it in a matrix and, then, determinants ( $Dx$ ,  $Dy$ , and  $D$ ). It is very quick and easy. This is the best way to solve linear equations from all the ways that I have learned. Cramer's rule is the easiest and best way.

Similarly, Arron said that "the method that I would choose first is Cramer's rule. Cramer's rule is easier and faster but it does involve more writing. Cramer's rule is easier to understand if you listen because it uses a matrix which means a lot of numbers." Also, Maria wrote that, in solving linear equations, she prefers Cramer's rule. "I prefer Cramer's rule for solving linear systems because it seems to be easier and less complicated than the methods before." In his journal writing, J'Rue said that:

...the easiest method for me is Cramer's rule. It is the fastest and easiest way to do linear system problems. I thought graphing was the easiest, but I'm wrong. If I had this method earlier, I would probably have a B or a C+. The steps are easy to do.

"To me Cramer's rule is way easier", Jasmine wrote: "All the other ways are a little complicated. But Cramer's rule saved my mathematics life. I easily understand how to solve linear systems now." "Between the methods of graphing, substitution, combination and Cramer's rule, I prefer to use Cramer's rule", Lucera wrote: "I don't clearly understand the other methods. I understand this one completely. It's way easier than the rest."

In terms of organization, Mauricio wrote that "another reason is because it's more organized and I learned the new word 'determinant'. Also, I learned that, when you use Cramer's rule, you have to use a matrix. That's why I prefer doing Cramer's rule instead of the rest." In terms of organization and error identification, Maria said that:

...the method is much clearer and neat for solving the problem. In Cramer's rule, it is much easier to identify an error in solving the problem. Plus the other methods took me more than a day to understand. Cramer's system shows clearly step by step how the problem is being solved.

Janet said that "I prefer to use Cramer's rule because it's so simple that you can easily be able to find the answers to the equation and, if you make a mistake, you can easily find out what you did wrong. Also, it is very easy to understand and solve." Eliseo included in his journal that "in my opinion, the easiest process to determine linear equations is Cramer's rule because it's simple to identify my mistakes.

Cramer's rule also helped me to unscramble the way to my answer and that's why Cramer's rule was the easiest for me." In her journal, Lateeana said: "I think that I prefer using Cramer's rule because it's easiest for me because you break the steps down little by little so that, if you do make a mistake, you would be able to detect it in your steps. So, out of the four methods, I prefer Cramer's rule." "If I were to compare all of the ways to find out a system of linear equations, I would say the easiest is Cramer's rule", said Stephanie. "It's so much easier than substitution, graphing, and combination. I think it's easier because it's a step-by-step thing and you can go back and it's easier to see where you messed up. I think that I am going to be using Cramer's rule until the end of the year."

The account of these students as written in their mathematics journals gave a strong argument on its own about the desirability of introducing this method in the curriculum early enough, especially in the middle-school algebra curriculum. The only disadvantage as written by the students is that it takes up more space on the paper because it uses more numbers and it does involve more writing. These accounts have demonstrated how the students have adopted this strategy for solving systems of linear equations in two variables and made it their own.

#### 5.4 ANALYSIS OF STUDENTS' WORK

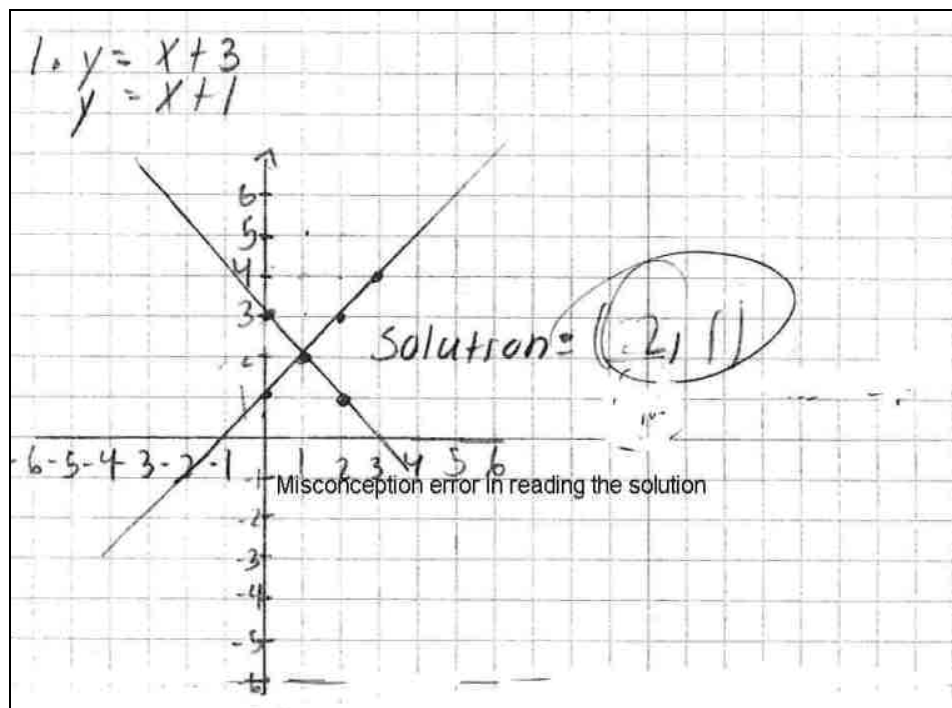


Figure 5.2. Student's Work Sample on Graphing Systems of Linear Equations

The problems solved by students during the lessons are presented in this section. The problems were versions of problems from standards-based algebra textbooks (e.g. Larson et al., 2001). Ashlock (1998, p. 40) noted that a student's work must not only be scored, but it must be analyzed if it is to provide useful information. Comprehensive past research on error analysis has been reported (Ashlock, 1998, pp. 312-315).

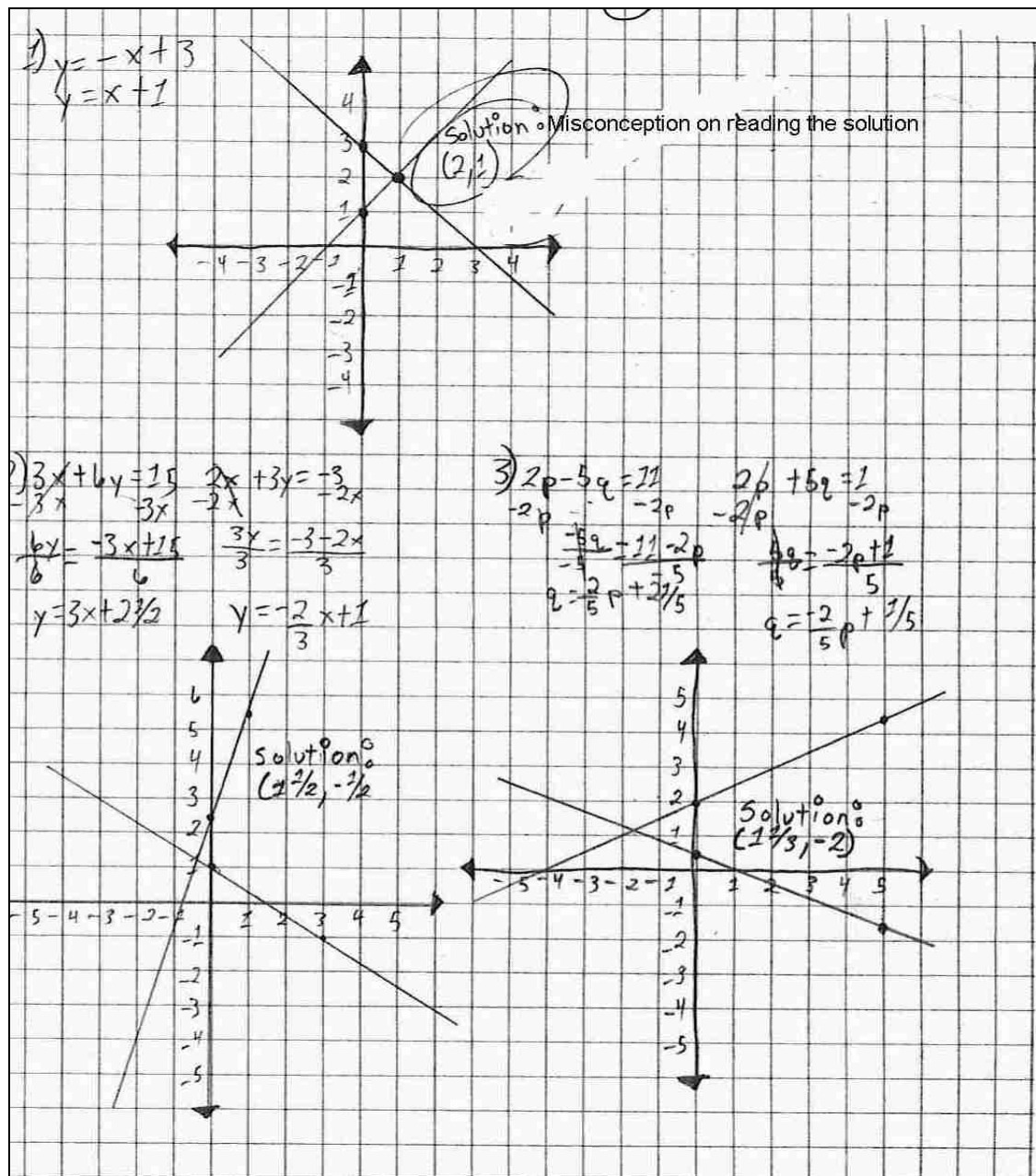




Figure 5.3. Student's Work Sample on Graphing Systems of Linear Equations  
Depicting Error

Figures 5.2 and 5.3 present students' work samples and identify misconceptions during the time when students were learning graphing methods. The method used in analyzing student work is similar to the approach used by Gallardo (2002) and Moschkovich (1999). These misconceptions were corrected in the class.

#### 5.4.1 Analysis of Students' Work on Substitution Method

The students were asked to respond to the following questions:

1. When solving a system of linear equations, how do you decide which variable to isolate in step 1 of the substitution method (Larson et al., 2001).
2. What four steps do you use to solve a system of linear equations by the substitution method?

The students' responses are presented below unedited:

*Student 1:*

You can decide by thinking about  $x$  and  $y$ . If you solve for  $x$  look at which ever problem has something like  $3x-2y=1$ . When solving for  $y$ , look for a problem like  $x-y=2$ .

Solve for  $y$  in the equation you chose in the exercise above:

$$3x + 2y - 3x = 7 - 3x$$

$$2y/2 = 4x/2$$

$$y = 2x$$

First, solve one of the equations for one of its variables. Second, substitute the expression from 1 into the other equation and solve for the other variable. Third, substitute the value from step 2 into the revised equation from step 1 and solve. Fourth, check the solution in the original equation.

*Student 2:*

To use the substitution method you must solve  $x$  and  $y$ . I need to solve a linear equation by substitution. Then solve the other equation for one of its variables. Substitute the expression from 1 into the other equation and solve for other variables. Then substitute the value in the solved equation and rewrite the equation and solve. Check to see if it is true.

*Student 3:*

The step I like to use to solve a system of a linear equation is substitution. Substitution is kind of easier to do because I use both equations at the same time. For example, I solve for  $x$  in equation 1 and, in equation 2, I substitute for what I got for  $x$  in the first equation.

These students' work sample analysis showed that they grasped the substitution method of solving systems of linear equations. Although most of the students explained that they would rather prefer not to use this method because they easily made mistakes solving problems involving the use of substitution method.

#### **5.4.2 Analysis of the Students' Work on Combination Method and Students' Journal**

The following question was given to the students as a learning journal response: "Explain how to solve systems of linear equations by the combination method." Students' responses to this writing prompt are catalogued below unedited:

##### *Student 1:*

First, you solve for  $x$  or  $y$  (it doesn't matter which is first). You solve when 2 numbers are the same numerically but one is negative and the other is positive. They cancel each other out. And you do inverse operation. If no numbers are the same, then you multiply the whole equation by 9 based on the coefficients you're trying the same. I like the combination because I learned it quickly because it's easy.

##### *Student 2:*

To solve this kind of method, you have to follow the following steps. First, you ask yourself if you have like coefficients. If so, you eliminate it or cross cancel to solve either  $y$  or  $x$ . If you are solving for  $x$ , you eliminate  $y$ . After that, you solve the other equation by eliminating  $x$  to solve for  $y$ . When doing all those steps, you make sure that your answers are right by checking them. You check by using one of the problems and replacing the  $x$  and  $y$  by the answer.

##### *Student 3:*

When solving a system of linear equations, there are three basic ways of solving it: graphing, substitution, and combination. In graphing, you convert to slope-intercept form and graph the equation. The point of intersection is your solution. For substitution, you solve for  $x$  in equation 1. Then, you substitute  $x$  in equation 2, this gives you the value of  $y$ . Substitute for  $y$  and solve for  $x$ . Combination (elimination) is my favorite method. You make sure that two lined up coefficients have the same number, but are opposites (e.g.  $3x$  and  $-3x$ ). These are eliminated. You add everything else and solve for  $x$  if you eliminated  $y$ , or solve for  $y$  if you eliminated  $x$ . Do both eliminations and you'll have the solutions. I like elimination better because, for some reason, it seems more natural to me, and easier.

##### *Student 4:*

To solve a linear system by using combination method, the first thing that you must do is get the same term but different signs on one of the variables. After that, you cross it out and add what you have for the other term and the

totals. Then you use inverse operation. When you get the answer for the first variable, you do the same for the other term.

*Student 5:*

To solve a problem using combination method, first you have to ask yourself if they are like coefficients. If there is you eliminate it or cross out  $y$  or  $x$ . If you're solving for  $x$ , eliminate  $y$ . After that, you solve the other equation by eliminating  $x$  to solve for  $y$ . Then you have your 2 answers. After, you have to check to see if it's correct.

*Student 6:*

Step 1: Arrange the equation with like terms in columns.

Step 2: Multiply, if necessary, the equations by numbers to obtain coefficients that are opposites for one of the variables.

Step 3: Add the equation from equation 2. Combining like terms with opposite coefficients will eliminate one variable.

Solve for the remaining variables.

Figure 5.4 shows the students' work samples involving the Combination Method. Looking at the students' work samples on the combination method, it was found that the majority of the students prefer to use this method of solving systems of linear equations than the substitution method because of the minimal procedural errors that they made. The students were able to attain a level of comfort after the initial errors were discovered and remediated.

Objective: SWBAT solve system of linear system by combination (Elimination) Period # 2-10-04

①  $3x + 6y = 15$   $\times 2 = 6x + 12y = 30$   
 $-2x + 3y = -3$   $\times 3 = -6x + 9y = -9$   
 $\frac{21}{21} = \frac{21}{21}$   
 $y = 1$

②  $-2x + 3y = -3$   
 $-2x + 3(1) = -3$   
 $-2x + 3 = -3$   
 $-2x = -6$   
 $\frac{-2x}{-2} = \frac{-6}{-2}$   
 $x = 3$  ✓

Check:  
 ①  $3x + 6y = 15$   
 $3(3) + 6(1) = 15$   
 $9 + 6 = 15$   
 $15 = 15$  ✓  
 ②  $-2x + 3y = -3$   
 $-2(3) + 3(1) = -3$   
 $-6 + 3 = -3$  ✓

---

③  $x + y = 4$   
 $x - y = -10$   
 $\frac{2x}{2} = \frac{-14}{2}$   
 $x = -7$   $\frac{6}{2} = 3$

④  $x - y = -10$   
 $-7 - y = -10$   
 $-7 - y = -10$   
 $+7$   
 $-y = -3$   
 $y = 3$

Check:  
 ①  $x + y = 4$   
 $-7 + 3 = 4$   
 $-4 = 4$  ✓  
 ②  $x - y = -10$   
 $-7 - 3 = -10$   
 $-10 = -10$  ✓

Obj: SWBAT solve system of linear equation by combination method Per 4, Feb-11-04

1)  $x + y = 4$   $1x + y = 4 \cdot 1$   
 $x - y = -10$   $1x - y = -10 \cdot -1$   
 $\frac{2x}{2} = \frac{-6}{2}$   
 $x = -3$   
 $x + y = 4$   
 $-x + y = 10$   
 $2y = 14$   
 $y = 7$

check  $-3 + 7 = 4$   
 $-3 - 7 = -10$   
 QED

2)  $3r - 2w = 1$   $3r - 2w = 1 \cdot -2$   
 $2r + 2w = 4$   $2r + 2w = 4 \cdot 3$   
 $5r = 5$   
 $r = 1$   
 $-4r + 4y = -2$   
 $+6r + 6y = 12$   
 $10y = 10$   
 $y = 1$

QED  
 $3(1) - 2(1) = 1$   $2(1) + 2(1) = 4$   
 $3 - 2 = 1$   $2 + 2 = 4$

Figure 5.4. Students' Work Samples on the Combination Method of Systems of

## Linear Equations

**5.4.3 Students' Work on Cramer's Rule**

In this section, students' work samples on solving systems of linear equations by Cramer's rule are presented. Cramer's rule uses a determinant to express the solution of a system of linear equations. Figures 5.5 and 5.6 depict some samples produced by the students who were involved in the innovative strategy involving learning systems of linear equations by Cramer's rule.

Students work samples on Cramer's rule method of solving systems of linear equations suggested that the students in the experimental group who were exposed to this method mastered the procedure and the concept behind this method. The work samples in particular show that the students did not make many calculation and procedural errors while using this approach to solve linear systems in two variables. In addition, their work samples show that they understood how to transform linear systems from standard form to matrix. They also learned how to find the determinants ( $D$ ), and subsequently solved the linear system by dividing  $D_y$  and  $D_x$  by  $D$ . This indicates that Cramer's rule can be learned in the middle-school as one of the methods for solving systems of linear equations.

Solve system of linear equation by Cramer's rule

8th period  
Feb. 20, 2004

1.  $3x - y = 9$   
 $x - y = 5$   $d(2, -3)$

Determinant  $\begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$D = \begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix} = -3 + 1 = -2$

$D_x = \begin{vmatrix} 9 & -1 \\ 5 & -1 \end{vmatrix} = -9 + 5 = -4$

$D_y = \begin{vmatrix} 3 & 9 \\ 1 & 5 \end{vmatrix} = 15 - 9 = 6$

$\frac{D_x}{D} = \frac{-4}{-2} = 2$  ✓  $\frac{D_y}{D} = \frac{6}{-2} = -3$  ✓

3)  $4x + 3y = 5$   
 $2x - 5y = 9$   $c(2, -1)$

Determinant  $\begin{bmatrix} 4 & 3 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$

$D = \begin{vmatrix} 4 & 3 \\ 2 & -5 \end{vmatrix} = -20 - 6 = -26$

$D_x = \begin{vmatrix} 5 & 3 \\ 9 & -5 \end{vmatrix} = -25 - 27 = -52 = \frac{-52}{-26} = 2$  ✓

$D_y = \begin{vmatrix} 4 & 5 \\ 2 & 9 \end{vmatrix} = 36 - 10 = 26 = \frac{26}{-26} = -1$  ✓

Figure 5.5. Students' Work Sample on Cramer's Rule Method of Systems of Linear Equations

obj: solve system of linear equations by Cramer's Rule algebra 1 part  
Matrix 2-20-04

1)  $\begin{cases} 3x - y = 9 \\ x - y = 5 \end{cases}$   $\begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

Determinant

$D_x = \begin{vmatrix} 9 & -1 \\ 5 & -1 \end{vmatrix} = -9 - (-5) = -4$

$D_y = \begin{vmatrix} 3 & 9 \\ 1 & 5 \end{vmatrix} = 15 - 9 = 6$

$D = \begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix} = -3 - (-1) = -3 + 1 = -2$

$x = \frac{-4}{-2} = 2$        $y = \frac{6}{-2} = -3$

$x = 2$        $y = -3$

---

3)  $\begin{cases} 4x + 3y = 5 \\ 2x - 5y = 9 \end{cases}$   $\begin{bmatrix} 4 & 3 \\ 2 & -5 \end{bmatrix} = 9$  Matrix

Determinant  $D_x = \begin{vmatrix} 5 & 3 \\ 9 & -5 \end{vmatrix} = -25 - 27 = -52$

$D_y = \begin{vmatrix} 4 & 5 \\ 2 & 9 \end{vmatrix} = 36 - 10 = 26$

Feb. 20, 2002

1.  $\begin{cases} 3x - y = 9 \\ x - y = 5 \end{cases}$

Matrix  $\begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$D_x = \begin{vmatrix} 9 & -1 \\ 5 & -1 \end{vmatrix} = -9 - (-5) = -9 + 5 = -4$

$D_y = \begin{vmatrix} 3 & 9 \\ 1 & 5 \end{vmatrix} = 15 - 9 = 6$

$D = \begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix} = -3 - (-1) = -3 + 1 = -2$

$x = \frac{D_x}{D} = \frac{-4}{-2} = 2$        $(2, -3)$

$y = \frac{D_y}{D} = \frac{6}{-2} = -3$

8th Period  
Feb. 20, 2004

Solve system of linear equation by Cramer's rule

1.  $3x - y = 9$   
 $x - y = 5$   $d(2, -3)$

Determinant  $\begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \end{bmatrix}$

$D = \begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix} = -3 + 1 = -2$

$D_x = \begin{vmatrix} 9 & -1 \\ 5 & -1 \end{vmatrix} = -9 + 5 = -4$

$D_y = \begin{vmatrix} 3 & 9 \\ 1 & 5 \end{vmatrix} = 15 - 9 = 6$

$\frac{D_x}{D} = \frac{-4}{-2} = 2$  ✓  $\frac{D_y}{D} = \frac{6}{-2} = -3$  ✓

3)  $4x + 3y = 5$   
 $2x - 5y = 9$   $C(2, -1)$

Determinant  $\begin{bmatrix} 4 & 3 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$

$D = \begin{vmatrix} 4 & 3 \\ 2 & -5 \end{vmatrix} = -20 - 6 = -26$

$D_x = \begin{vmatrix} 5 & 3 \\ 9 & -5 \end{vmatrix} = -25 - 27 = -52 = \frac{-52}{-26} = 2$  ✓

$D_y = \begin{vmatrix} 4 & 5 \\ 2 & 9 \end{vmatrix} = 36 - 10 = 26 = \frac{26}{-26} = -1$  ✓

Figure 5.6. Students' Work Samples on the Cramer's Rule Method of Solving Systems of Linear Equations

## 5.5 SUMMARY OF QUALITATIVE FINDINGS

This chapter reported the findings from the use of qualitative data-gathering methods. The qualitative information was collected using audiotaped interviews, students' journal writings, and analysis of students' work samples. Qualitative data-gathering was important and complemented quantitative data-gathering. Patterns in the qualitative information, based on reflective journal entries, student audiotaped interviews, and analysis of student work samples are summarized below:



*Finding 1: Classroom reflective journals, student interviews, and analysis of student work samples each substantiated the quantitative findings of an increase in involvement, shared control, task orientation, and student negotiation during the implementation of the innovative strategy for teaching and learning systems of linear equations.*

*Finding 2: Classroom reflective journals, student interviews, and analysis of student work samples supported the quantitative results of larger improvements in perceptions of classroom involvement, attitudes towards mathematics and mathematics achievement for the experimental group than for the control group.*

*Finding 3: The students who were involved in the experimental group showed increased attitudes to mathematics in general. This is particularly evident in the students' interviews and students' work samples on the Cramer's rule method of solving systems of linear equations. This somewhat substantiates the findings from quantitative data-gathering methods.*

The use of qualitative methods has been reported for investigating students' understanding in terms of whether they involve naturalistic settings, interviews, conceptual relationships, diagnostic test items, or computerized diagnosis and methods that draw on ethnographic techniques such as participant observation (Burns, 1997; Duit et al., 1996). Spinner and Fraser (2005) and De Bock, Van Dooran, Janssens, and Verschaffel (2002) are some of the recent researchers in mathematics who have used interview sessions. Analyses of the qualitative data from students' interviews, students' learning journal writings, and students' work helped to answer the research questions (discussed in Chapters 1 and 4) in order to triangulate the findings from the quantitative data.

The interview script and students' learning journal writing suggested students' preference for and easy understanding of the Cramer's rule.

The students' journal entries suggested that introducing Cramer's rule as a method for solving systems of linear equations in the middle-school enhanced their enjoyment of mathematics and gave them the impression that their classroom

environment was task oriented and involved all students. The use of a mixed-method approach for collecting classroom environment data enhanced the validity of the findings because a range of methods which each has strengths and weakness on its own was utilized.

Whereas Chapter 4 reported the quantitative data-gathering methods and findings, this chapter reports the qualitative data-gathering methods which complemented the quantitative data. The following chapter provides a discussion of the findings, as well as information regarding the significance and limitations of the study and suggestions for future research.

## **Chapter 6**

### **DISCUSSION, SIGNIFICANCE AND CONCLUSIONS**

#### **6.1 OVERVIEW**

This chapter presents a discussion of the significance, findings, limitations and conclusions of my research. It also identifies desirable directions for future research. The chapter is organized according to the following topics:

- Summary of research methods (Section 6.2)
- Summary of results and findings (Section 6.3)
- Significance (Section 6.4)
- Limitations and suggestions for future research (Section 6.5)
- Conclusions (Section 6.6).

#### **6.2 SUMMARY OF RESEARCH METHODS**

My research investigated whether the use of an innovative method which involves a numerical method (Cramer's rule) for teaching and learning systems of linear equations enhanced the classroom environment, students' attitudes, and students' conceptual development compared to students who were taught by a traditional approach. Two subgroups of students were compared in the study (experimental and control groups). The experimental group experienced the innovative strategy and the control group experienced a traditional approach.

The students who participated in this study were eighth grade middle-school students from a low socio-economic community. The ethnogeographic distribution of the students was about 46% African American, 51% Hispanic and 3% other. The students' ages ranged between 13 and 14 years. A total of 661 students responded to the learning environment and attitude surveys. These data were used to answer research questions about the validity of the instruments, about associations between the classroom environment and outcomes among middle-school students in

California, and about gender differences. A subsample of 101 students responded to achievement and concept development questions in addition to the classroom environment and attitude surveys. This sample was divided into two subgroups of 61 students (experimental) and 40 students (control).

The methodology that I employed involved collecting comparative data about the effectiveness of the innovative strategy, which was assessed using achievement tests, and questionnaire scales which were administered as pretests and posttests during an academic year. Three scales of Personal Relevance, Shared Control, and Student Negotiation from the Constructivist Learning Environment Survey (CLES) and three scales of Involvement, Task Orientation, and Investigation from the What Is Happening In this Class? (WIHIC) questionnaire provided quantitative data about students' perceptions of their classroom learning environment (Fraser, 1998b). The two scales of Normality of Mathematicians and Enjoyment of Mathematics, selected from the Test of Mathematics-Related Attitudes (TOMRA), and achievement tests provided information about students' attitudes towards mathematics and conceptual development of algebra skills. The quantitative data provided important information about the innovative method that can be adopted and utilized to improve the learning environment in a way that will enhance the students' achievement and attitudes.

In order to determine the validity of CLES, WIHIC, and TOMRA when used with middle-school students in California, principal components factor analyses with varimax rotation were used to furnish evidence about the structure of the questionnaires for assessing classroom environments and attitudes among middle-school mathematics students. The Cronbach alpha coefficient was computed for two units of analysis (individual and class mean) for each scale of the CLES, WIHIC, and TOMRA in order to estimate the internal consistency reliability (extent to which items within a scale assess the same construct). The discriminant validity (extent to which a scale measures a distinct construct that is not assessed by the other scales) of each questionnaire was determined by calculating the mean correlation of each scale with the other scales. An ANOVA was also used to determine the ability of each CLES and WIHIC scale to differentiate between the perceptions of students in different classes.

Simple correlation and multiple correlation analyses were performed to determine associations between classroom environment perceptions and students' attitudes. These associations were calculated for two units of statistical analysis, namely, the student and the class mean.

Effect sizes and *t*-tests for paired sample were calculated to determine changes in classroom environment perceptions, attitudes, and achievement for both the experimental and control groups. ANCOVA was calculated as well to determine differential pretest-posttest changes experienced by the experimental and control groups in classroom environment perceptions, attitudes, and achievement.

MANOVA for repeated measures was performed to determine gender differences in learning environment perceptions, attitudes to mathematics, and achievement. The unit of analysis for gender comparisons was the student.

Qualitative information, gathered through audiotaped interviews, students' journal, and analysis of students' work was used to clarify students' opinions about the new approach, their classroom environment perceptions, attitudes, and conceptual development. These qualitative information-gathering tools were utilized to obtain more in-depth understanding of the learning environments (Tobin, Kahle, & Fraser, 1990) and the results of our study (Punch, 1998, pp. 149-150), as well as insights into students' perceptions (Spinner & Fraser, 2005). The responses from the students' interviews and students' reflective journals from the group that experienced the innovative methods generally suggested that introducing Cramer's rule as a method for solving systems of linear equations in the middle school can be beneficial and therefore might be considered for inclusion in the middle school Algebra 1 curriculum more widely in California. Using only quantitative data would not have provided the richness that was derived from using mixed methods (Johnson & Onwuegbuzie, 2004).

Whereas this section summarized the methods from the study, the results and findings from this study are summarized in the next section.

### 6.3 SUMMARY OF RESULTS AND FINDINGS

Analyses of data obtained from the environment and attitude survey instruments, achievement tests, and qualitative data-gathering methods helped to answer the following research questions:

#### Research Question #1

*Are questionnaires for assessing classroom environments and attitudes to mathematics valid when used with middle-school students in California?*

#### Research Question #2

*Is an innovative teaching approach – involving the use of information technology, numerical methods (Cramer’s rule), and constructivist methods for the topic of systems of linear equations – effective in terms of promoting:*

- (a) a positive classroom environment*
- (b) student attitudes to mathematics*
- (c) student achievement and ability to identify and apply concepts?*

#### Research Question #3

*Are there associations between classroom environment and student attitudes to mathematics?*

#### Research Question # 4

*Are there gender differences in perceptions of classroom environments, attitudes to mathematics, and mathematics achievement?*

In terms of the validity of the CLES, WIHIC, and TOMRA when used with middle-school students in California, the factor analysis results reported in Tables 4.1 to 4.3 attest to the sound factor structure of each questionnaire. The results for each CLES, WIHIC, and TOMRA scale for the alpha reliability and discriminant validity (mean correlation with other scales as a convenient index) for two units of analysis (individual and class mean) compare favorably with the results for other well-established classroom environment instruments (see Fraser, 1998b; Dorman, 2003). A one-way analysis of variance (ANOVA) was calculated for each scale of the CLES and WIHIC to investigate its ability to differentiate between the perceptions of

students in different classrooms. The ANOVA results suggest that students perceived the learning environments of different mathematics classrooms differently on CLES and WIHIC scales.

The results of simple correlation and multiple correlation analyses of attitude-environment associations for two units of analysis clearly indicated that there is an association between the learning environment and students' attitudes for this group of middle-school mathematics students. Specifically, there is a positive and statistically significant correlation between Normality of Mathematicians and Student Negotiation, Involvement, and Task Orientation with the individual as the unit of analysis. There is also a positive and statistically significant correlation between Enjoyment of Mathematics and all three CLES and three WIHIC scales with the student as a unit of analysis, and for the four scales of Personal Relevance, Shared Control, Involvement, and Task Orientation with the class mean as the unit of analysis. The multiple correlation between the group of three CLES and three WIHIC scales and each of the two TOMRA scales is statistically significant for the individual as a unit of analysis.

Effect sizes and *t*-tests for paired sample were used to determine changes in classroom environment perceptions, attitudes, and achievement for the experimental and control groups. The results obtained for pretest-posttest differences on each scale reveal that there are statistically significant differences ( $p < 0.05$ ) on: the CLES scale of shared control for the experimental group, the TOMRA scale of Normality of Mathematicians for both the control and the experimental groups, the TOMRA scale of Enjoyment of Mathematics for the experimental group, and the achievement measure for both groups. Also ANCOVA was calculated to determine differential changes experienced by the experimental and control groups in classroom environment perceptions, attitudes, and achievement. The results suggest that there are a statistically significant differential changes for Task Orientation, Normality of Mathematicians, Enjoyment of Mathematics, and achievement between the experimental and control groups. In each case, larger pretest-posttest changes were experienced by the experimental group than the control group.

A two-way MANOVA with repeated measures on one factor was utilized to investigate gender differences in terms of students' perceptions of classroom environment and attitudes to mathematics, as well as mathematics achievement. A statistically significant but small difference was found between the genders for Student Negotiation and Task Orientation. Female students perceived their mathematics classrooms somewhat more positively than did the male students. There was no statistically significant difference between the genders on achievement and students' attitudes to mathematics.

The findings from using quantitative data-gathering methods in my study involving evaluation of an innovative strategy for teaching and learning systems of linear equations in terms of classroom environment, students' attitudes, and conceptual development are summarized below.

- *The CLES, WIHIC, and TOMRA each displayed satisfactory factorial validity. The total percentage of variance extracted was 50.73% for the three CLES scales, 48.65% for the three WIHIC scales, and 32.08% for the two TOMRA scales.*
- *The CLES, WIHIC, and TOMRA demonstrated satisfactory internal consistency reliability for two units of analysis (individual and class mean).*
- *Discriminant validity results (using the mean correlation of a scale with other scales as a convenient index) for two units of analysis show that raw scores on scales overlap, but not to the extent that the psychometric structure of the instruments is violated. Moreover, the factor analysis results support the independence of factor scores on the scales in each instrument.*
- *The CLES and WIHIC scales can differentiate significantly between the perceptions of students in different classrooms.*

In order to answer the second research question, changes in pretest and posttest scores were analyzed separately for 61 students in an experimental group and 40 students in a control group using effect sizes and *t*-tests for paired samples. These



analyses were conducted to investigate if an innovative teaching approach – involving the use of numerical methods (Cramer’s rule) and constructivist methods for the topic of systems of linear equations – was effective in terms of promoting (a) a positive classroom environment, (b) student attitudes to mathematics, and (c) student achievement and student ability to identify and apply concepts. The main findings are listed below:

- *Statistically significant pretest-posttest differences were evident for:*
  - *the CLES scale of Shared Control for the experimental group*
  - *the TOMRA scale of Normality of Mathematicians for both the experimental and control groups*
  - *the TOMRA scale of Enjoyment of Mathematics for the experimental group*
  - *open-ended achievement for both the experimental and control groups.*
- *In general, relative to the control group, the experimental group showed somewhat larger changes for the classroom environment and attitudes scales, as well as open-ended achievement. The effect sizes in standard deviations for pretest-posttest changes were 0.31 (experimental) and 0.05 (control) for Shared Control, 1.99 (experimental) and 1.11 (control) for Normality of Mathematicians, 0.61 (experimental) and 0.21 (control) for Enjoyment of Mathematics, and 1.80 (experimental) and 1.11 (control) for achievement.*

ANCOVA was also conducted to determine if differential changes were experienced by the experimental and control groups by examining posttest classroom environment perceptions, attitudes, and achievement scores when pretest scores on the corresponding scale is held constant (covariate):

- *Relative to the control group, the experimental group experienced statistically significantly larger pretest-posttest changes in terms of the*

*WIHIC scale of Task Orientation, the TOMRA scales of Normality of Mathematicians and Enjoyment of Mathematics, and achievement.*

The findings for the third research question involving investigating the associations between the classroom environment and student attitudes to mathematics are summarized below. This was determined using simple correlation and multiple regression analyses for two units of analysis:

- *A positive and statistically significant correlation exists between Normality of Mathematicians and the CLES scale of Student Negotiation and the WIHIC scales of Involvement and Task Orientation with the individual as the unit of analysis.*
- *There is a positive and statistically significant correlation between Enjoyment of Mathematics and all three CLES and three WIHIC scales with the student as a unit of analysis, and for the four scales of Personal Relevance, Shared Control, Involvement, and Task Orientation with the class mean as the unit of analysis.*
- *The multiple correlation between the group of three CLES and three WIHIC scales and each of the two TOMRA scales is statistically significant for the individual as a unit of analysis.*
- *Overall, improved student attitudes are associated with more emphasis on all of the aspects of constructivism, especially Personal Relevance and Shared Control, as well as the WIHIC scales of Task Orientation and Involvement.*

Finally, MANOVA was performed in order to answer the fourth research question concerning gender differences in perceptions of classroom environments, attitudes to mathematics and achievement. The findings are summarized below:

- *A statistically significant but small difference was found between the genders for Student Negotiation and Task Orientation. Female students perceived their mathematics classroom somewhat more positively than the male*

*students for this group of Californian mathematics students. There was no statistically significant difference between the genders on achievement or students' attitudes to mathematics.*

Whereas the proceeding paragraphs highlighted the findings from the quantitative data-gathering methods, the findings from the qualitative data-gathering methods discussed in Chapter 5 are important and complemented quantitative data-gathering. Patterns obtained from the qualitative information, based on reflective journal entries, student audiotaped interviews, and analysis of student work samples are summarized below:

- *Classroom reflective journals, student interviews, and analysis of student work samples each substantiated the quantitative findings of increased involvement, shared control, task orientation, and student negotiation during the implementation of the innovative strategy for teaching and learning systems of linear equations.*
- *Classroom reflective journals, student interviews, and analysis of student work samples supported the quantitative results of larger improvements in perceptions of classroom involvement, attitudes towards mathematics and mathematics achievement for the experimental group than for the control group.*

The findings from this study (quantitative and qualitative) generally suggest that this innovation in teaching and learning systems of linear equations is a worthwhile effort. Students who were involved in this innovative teaching method often indicated a preference for the use of Cramer's rule, which is a numerical approach to solving systems of linear equations. As well, students' interview scripts and learning journal writings indicated their preferences and easy understanding of Cramer's rule. Overall, my findings support the wider introduction of this method for teaching and learning of systems of linear equations in the middle schools' curriculum in California.

#### **6.4 SIGNIFICANCE**

This study can be viewed as educationally important and significant for several reasons. Firstly, although the survey instruments utilized in this study are well-established, my research still cross-validated them independently before using them to evaluate an innovative educational approach with middle-school students in California.

Secondly, my study is important educationally in that it evaluated a stimulating classroom environment that makes use of variety and interesting strategies for learning, such as computer-assisted approaches. The teaching and learning of systems of linear equations involving a numerical method (Cramer's rule) has never been undertaken before in middle schools in California. Also, these topics have not been considered for inclusion in the middle-school algebra curriculum in California. In order to help to prepare students conceptually to be adaptable for college learning and challenges, an attempt was made to incorporate innovative mathematics teaching and learning into the middle-school curriculum early enough to accommodate the students' learning needs.

Thirdly, this research is important also in that there is only a relatively small number of learning environment studies internationally that has focused specifically on mathematics classes (e.g. Dorman, 2001; Majeed et al., 2002; Mink & Fraser, 2005; Raaflaub & Fraser, 2003; Sebela et al., 2004; Spinner & Fraser, 2005; Taylor et al., 1994). None of this handful of learning environment studies has focused primarily on the teaching and learning of systems of linear equations.

Fourthly, the use of a control group in this research design generated data that could be used to meaningfully compare the group who experienced the innovative strategy for teaching and learning systems of linear equations (experimental group) with another teaching method. Also both the experimental and control groups were given pretest and posttests in order to measure changes in classroom environment, students' attitudes, and conceptual development over a time period. This pre-posttest design is useful for evaluating new educational programs (Teh & Fraser, 1994).

Finally, this study is distinct in that it did not rely only on achievement. Student attitudes to mathematics and their classroom environment perceptions also were

main foci. Recent reports and standardized test scores have shown that California algebra students are not performing proficiently. Therefore, the results of the study are likely to provide information to help in the process of improving achievement, attitudes, and classroom environments among eighth-grade algebra students in California.

## 6.5 LIMITATIONS AND SUGGESTIONS FOR FUTURE RESEARCH

While my study has some strengths, it is very important also to point out its limitations. First, the statistical power is limited in some data analyses due to the small sample size of 101 students available for the comparison of experimental and control groups. Moreover, because of the smallness of the sample size for the comparison of experimental and control groups, it was not possible to use MANOVA and MANCOVA. My use of multiple *t*-tests and multiple ANCOVAs could have given rise to Type I errors. The qualitative component of this study also was limited due to small number of students ( $N=12$ ) who were interviewed. Although conducting extensive and comprehensive qualitative data collection would have been preferable, nonetheless, an attempt was made to obtain students' journal writing to complement this shortcoming.

In future research, therefore, I suggest the use of larger, more diverse and more representative samples that will permit greater confidence in and wider generalizability of the findings, as well the opportunity to use MANOVA and MANCOVA in analyzing the data.

Secondly, the representativeness of the sample could be another limiting factor in that, when compared to the general eighth-grade population in California, my sample could be considered neither a sizeable fraction of the population nor representative of the full range of schools and students. This limits the generalizability of findings. Student cultural background is another variable that was not considered in this research. Some students are African American, while some are limited English proficient Hispanic students who might not be in tune with the constructivist approach of teaching and learning because of their English language deficiencies – which the innovative method incorporates. For instance, the sociocultural

background of most of the students in my school community might make it difficult for them to adapt to the new teaching and learning approach. Also, the findings from this study might have limited generalizability to other cultural backgrounds.

I suggest that future researchers replicate this study with samples that represent a range of sociocultural groups and socioeconomic status in order to obtain more generalizable results.

Thirdly, the teacher being the researcher in this study could have given rise to bias and errors. The researcher might not have been an impartial observer. The ‘halo effect’ is a form of researcher bias commonly more prevalent at the data analysis stage. It occurs when a researcher is scoring open-ended responses and allows his or her prior knowledge of or experience with the participants to influence the scores given (Onwuegbuzie & Daniel, 2003). It was not always possible to have other teachers of mathematics observe the experimental group classes during the study. If this were possible, their feedback and critique would have been valuable in enhancing the quality of this study. Seeing what the researcher wants to see could have obscured his sound judgment, especially in qualitative data gathering. Spinner and Fraser (2005) encountered and reported this problem in their study.

In order to avoid the halo effect and researcher bias, it is suggested that future researchers involve other experts in scoring open-ended and constructive responses, as well as consulting with different experts to observe and give feedback on the performance of the group and the researcher during the study.

## **6.6 CONCLUSIONS**

This chapter summarized the methods, results, and limitations of my evaluation of an innovative strategy for teaching and learning systems of linear equations in term of classroom environment, students’ attitudes, and conceptual development. My research was conducted among eighth-grade middle-school students in a low socio-economic area in California. Overall, this concluding chapter ties together the study in terms of discussing its research findings and educational importance.

Factor analysis attested to the sound scale structure of the CLES, WIHIC, and TOMRA when used with middle-school students in California. The results for each scale's alpha reliability and discriminant validity compared favorably with the results of past studies. The classroom environment scales used in my study were able to differentiate between the perceptions of students in different classrooms.

The ANCOVA results suggested that, relative to the control group, the experimental group experienced statistically significantly larger pretest-posttest changes for Task Orientation, Normality of Mathematicians, Enjoyment of Mathematics, and achievement. Overall, a comparison of the pretest-posttest changes for the experimental group which experienced the innovative strategy, with those for a control group, supported the efficacy of the innovative teaching methods in terms of learning environment perceptions, attitudes to mathematics, and mathematics concept development.

My study revealed associations between the learning environment and students' attitudes to mathematics for this group of middle-school mathematics students. In particular, my study suggested that more positive student attitudes are associated with more emphasis on all of the aspects of constructivism as assessed by the CLES.

A two-way MANOVA with repeated measures on one factor was utilized to investigate gender differences in terms of students' perceptions of classroom environment and attitudes to mathematics, as well as mathematics achievement. A statistically significant but small difference was found between the genders for Student Negotiation and Task Orientation. Female students perceived their mathematics classrooms somewhat more positively than did the male students. There was no statistically significant difference between the genders on achievement and students' attitudes to mathematics.

Qualitative information, gathered through audiotaped interviews, students' journal, and analysis of students' work from the group that experienced the innovative methods, generally supported the quantitative findings. The qualitative findings suggested that introducing Cramer's rule as a method for solving systems of linear equations in the middle school can be beneficial and therefore might be considered

for inclusion in the middle-school Algebra 1 curriculum more widely in California. The quantitative and qualitative data tentatively supported the effectiveness of the innovative method in providing a positive classroom learning environment.

My study tentatively suggests the desirability of introducing this method for the teaching and learning of systems of linear equations in the middle-school curriculum in California. Therefore, my research somewhat supports teaching eighth-grade students rigorous and challenging mathematics topics irrespective of their socio-cultural and socio-economic status. If the students from a low socio-economic community who participated in this study could learn the intended topic (systems of linear equations involving Cramer's rule) and their attitudes towards mathematics improved as a result of the instructional approach, then other students in middle and higher socio-economic communities also could benefit from this approach.

I therefore recommend that a study be undertaken in order to replicate my research to add confidence to and validate my findings. As suggested in Section 6.5, this could be undertaken with a larger sample involving more schools in California and other states. It is recommended also that this study be extended beyond the shores of the United States to include an international sample, which could yield results and findings for comparisons.



## References

- Adams, J. (2000, April). *Development of an instrument to evaluate school-level environments in the special sector: The Special School-Level Environment Questionnaire*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Aldridge, J. M., & Fraser, B. J. (2000). A cross-cultural study of classroom learning environments in Australia and Taiwan. *Learning Environments Research*, 3, 101-134.
- Aldridge, J. M., Fraser, B. J., & Huang, T. C. I. (1999). Investigating classroom environments in Taiwan and Australia with multiple research methods. *Journal of Educational Research*, 93, 48-62.
- Aldridge, J. M., Fraser, B. J., Taylor, S., & Chen, C. C. (2000). Constructivist learning environments in a cross-national study in Taiwan and Australia with multiple research methods. *International Journal of Science Education*, 22, 37-55.
- Anderson, G., & Arsenault, N. (1998). *Fundamentals of educational research*. London: Falmer Press.
- Anderson, G. J., & Walberg, H. J. (1974). Learning environments. In H.J. Walberg (Ed.), *Evaluating educational performance: A sourcebook of methods, instruments and examples* (pp. 81-98 ). Berkeley, CA: McCutchan.
- Ashloch, R. B. (1998). *Error patterns in computation* (7<sup>th</sup> ed.). Upper Saddle River, NJ: Prentice-Hall.
- Bahrack, H. P., & Hall, L. K. (1991). Lifetime maintenance of high school mathematics content. *Journal of Experimental Psychology: General*, 120, 22-23.
- Bell, A., & Janvier, C. (1981). The interpretation of graphs representing situations. *For the Learning of Mathematics*, 2, 34-42.
- Bergen, D. (2003). Choosing technology to meet varied learning purposes. *Childhood Education*, 27, 114-5
- Blose, R. J., & Fisher, D. (2003, April). *Effects of teachers' school-level environment perceptions on changing elementary mathematics classroom environments*. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.
- Boaler, J. (2000). Mathematics from another world: Traditional communities and the alienation of learners. *The Journal of Mathematical Behavior*, 18, 379-397.
- Bock, R. D. (Ed.). (1989). *Multilevel analysis of educational data*. San Diego, CA: Academic Press.
- Brooks, M. G., & Brooks, J. B. (1993). *In search of understanding: The case for constructivist classrooms*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Brooks, M. G. & Brooks, J. B. (1999). The courage to be constructivist. *Educational Leadership*, 57, 18-24.
- Brown, J. S., & Burton, R. B. (1978). Diagnostic models for procedural bugs in basic mathematics skills. *Cognitive Science*, 2, 155-192.
- Brown, J. S., & VanLehn, K. (1980). Repair theory: A generative theory of bugs in procedural skills. *Cognitive Science*, 4, 379-426.
- Bryk, A. S., & Raudenbush, S. W. (1992). *Hierarchical linear models: Applications and data analysis methods*. Newbury Park, CA: Sage Publications.

- Burden, R., & Fraser, B. J. (1993). Use of classroom environment assessments in school psychology: A British perspective. *Psychology in the Schools*, 30, 232-240.
- Burns, R. (1997). *Introduction to research methods* (3rd ed.). Melbourne: Longman.
- Burstein, L., Linn, R. L., & Capell, F. J. (1978). Analyzing multilevel data in the Presence of heterogeneous within-class regressions. *Journal of Educational Statistics*, 3, 347-383.
- Burton, R. B. (1982). DEBUGGY: Diagnosis errors in basic mathematical skills. In D. Sleeman & J. S. Brown (Eds.), *Intelligent tutoring systems*. New York: Academic Press.
- Bush, W. S., Brown, E. T., Ronau, R., Myers, M., McGatha, M., Thompson, C., Moody, V., Karp, K. (2005, April). *Reliability and validity of diagnostic mathematics assessments for middle school teachers*. Paper presented at the annual meeting of the American Education Research Association, Montreal, Canada.
- Bush, W. S., & Leinwand, S. (Eds.) (2000). Classroom assessment for school mathematics. *Mathematics assessment: A practical handbook for Grades 6–8*. Reston, VA: National Council of Teachers of Mathematics.
- California Department of Education. (2000). *Mathematics framework for California public schools, Kindergarten through Grade twelve*. Sacramento, CA: Department of Education.
- Carpenter, T., Corbitt, M., Kepner, H., Lindquist, M., & Reys, R. (1982). Student performance in algebra: Results from the national assessment. *School Science and Mathematics*, 82, 514-531.
- Castillo, G. E., Peiro, M. M., & Fraser, B. J. (2005, August). *Grade-level, gender and ethnic differences in attitudes and learning environment in high school mathematics*. Paper presented at the Fourth International Conference on Science, Mathematics and Technology Education, BC, Canada.
- Cattell, R. B. (1966). The scree test for the number of factors. *Multivariate Behavioral Research*, 1, 245-276.
- Chen, H., Chang, W., & Chang, H. (2002). *Different gender students' perceptions of classroom climate in a trial of a teacher-developed interdisciplinary module*. *Proceeding of National Science Council*, 12 (3), 79-90.
- Cheung, K.C. (1993). The learning environment and its effects on learning: Product and process modeling for science achievement at the sixth form level in Hong Kong. *School Effectiveness and School Improvement*, 4, 242-264.
- Chiappini, G., & Bottino, R. M. (2001). *Visualisation in teaching-learning mathematics: The role of the computer*. Genova, Italy: Istituto per la Matematica Applicata.
- Chionh, Y. H., & Fraser, B. J. (1998, April). *Validation of the 'What Is Happening In This Class' questionnaire*. Paper presented at the annual meeting of National Association for Research in Science Teaching, San Diego, CA.
- Ciochine, J., & Polivka, G. (1997). The missing link? Writing in mathematics class! *Mathematics Teaching in the Middle School*, 2, 316-20.
- Clark, C. M., & Peterson, P. L. (1986). Teachers' thought processes. In M. Wittrock (Ed.), *Handbook of research on teaching* (3<sup>rd</sup> ed; pp. 255-296) New York: Macmillan.
- Clement, J. (1980, April). *Algebra word problem solutions: Analysis of a common misconception*. Paper presented at the annual meeting of American Educational Research Associations, Boston.

- Clement, J. (1982). Algebra word solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education*, 13, 16-30.
- Coakers, S. J., & Steed, L. G. (1996). *SPSS for windows: Analysis without anguish*. New York: John Wiley.
- Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R.B. Davis, C.A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (Monograph Number 4, pp. 125-146). Reston, VA: National Council of Teachers of Mathematics.
- Cohen, D. K., & Ball, D. L. (1990). Relations between policy and practice: A commentary. *Educational Evaluation and Policy Analysis*, 12, 331-338.
- Conte, S. D., & de Boor, C. (1980). *Elementary numerical analysis*. New York: McGraw-Hill.
- Cooper, G., & Sweller, J. (1987). Effects of schema acquisition and rule automation On mathematical problem-solving transfer. *Journal of Educational Psychology*, 79, 347-362.
- Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1998). Qualitatively different experiences of learning mathematics at university. *Learning and Instruction*, 8, 455-468.
- Cronbach, L. J. (1951). Coefficient alpha and the internal structure of tests. *Psychometrika*, 16, 297-335.
- Crowley, M. (1993). Student mathematics portfolio: More than a display case. *Mathematics Teacher*, 86, 544-547.
- Dart, B., Burnett, P., Boulton-Lewis, G., Campbell, J., Smith, D., & McCrindle, A. (1999). Classroom learning environments and students' approaches to learning. *Learning Environments Research*, 2, 137-156.
- De Bock, D., Van Dooran, W., Janssens, D., & Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of secondary school students' errors. *Educational Studies in Mathematics*, 50, 311-334.
- Denzin, N. K., & Lincoln, Y. S. (1994). Introduction: Entering the field of qualitative research. In N.K. Denzin & Y.S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 1-17). Newbury Park, CA: Sage.
- DiPillo, M. L., Sovchik, R., & Moss, B. (1997). Exploring middle graders' mathematical thinking through journals. *Mathematics Teaching in the Middle School*, 2, 308-14.
- Dorman, J. P. (2001). Associations between classroom environment and academic efficacy. *Learning Environments Research*, 4, 243-257.
- Dorman, J. (2003). Cross-national validation of the What is Happening in This Class? (WIHIC) questionnaire using confirmatory factor analysis. *Learning Environments Research*, 6, 231-245.
- Dorman, J., Fraser, B. J., & McRobbie, C. J. (1994, April). *Rhetoric and reality: A study of classroom environment in Catholic and government secondary schools*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Duit, R., & Confrey, J. (1996). Reorganizing the curriculum and teaching to improve learning in science and mathematics. In D.F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics* (pp. 79-93). New York: Teacher College Press.

- Duit, R. Treagust, D. F., & Mansfield, H. (1996). Investigating students' understanding as a prerequisite to improving teaching and learning in science and mathematics. In D.F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics* (pp. 17-31). New York: Teacher College Press.
- Dreyfus, T. (1995). Imagery for diagrams. In R. Sutherland, & J. Mason (Eds.), *Exploiting mental imagery with computer in mathematics education* (pp. 3-19). Berlin: Springer Verlag.
- Dryden, M., & Fraser, B. J. (1996, April). *Evaluating urban systemic reform using classroom learning environment instruments*. Paper presented at the annual meeting of the American Educational Research Association, New York.
- Dryden, M., & Fraser, B. J. (1998, April). *The impact of systemic reform efforts on instruction in high school classes*. Paper presented at the annual meeting of the American Educational Research Association, San Diego, CA.
- Entwistle, N., & Ramsden, P. (1983). *Understanding student learning*. London: Croom Helm.
- Erickson, F. (1998). Qualitative methods in research for science education. In B. J. Fraser and K. G. Tobin (Eds.), *The international handbook of science education* (pp. 1155-1173). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Evans, D. L., Midkiff, C., Morgan, J., Krause, J. M., Notaros, B. M., Rancour, D., & Wage, K. (2001, November). *Tools for assessing conceptual understanding in the engineering sciences*. Paper presented at 32<sup>nd</sup> ASEE/IEEE Frontiers in Education Conference, Boston, MA.
- Fang, Z. (1996). A review of research on teacher beliefs and practices. *Educational Research*, 38, 47-65.
- Felsine, J. (1983). Concept mapping: A method for detection of possible student misconceptions. In H. Helm, & J. D. Novak (Eds.), *Proceedings of the International Seminar on Misconceptions in Science and Mathematics* (pp. 467-476). Ithaca, NY: Department of Education, Cornell University.
- Ferguson, P. D., & Fraser, B. J. (1996). *Schools size, gender, and changes in learning environment perceptions during the transition from elementary to high school*. Paper presented at the annual meeting of the American Educational Research Association, New York.
- Ferguson, P., & Fraser, B. J. (1999). Changes in learning environment during the transition from primary to secondary school. *Learning Environments Research: An international Journal*, 1, 369-383.
- Fisher, D. L., & Fraser, B. J. (1981). Validity and use of My Class Inventory. *Science Education*, 65, 145-156.
- Fisher, D. L., & Fraser, B. J. (1983a). A comparison of actual and preferred classroom environment as perceived by science teachers and students. *Journal of Research in Science Teaching*, 20, 55-61.
- Fisher, D. L., & Fraser, B. J. (1983b). Validity and use of classroom environment scale. *Educational Evaluation and Policy Analysis*, 5, 261-271.
- Fisher, D. L., Fraser, B. J., & Rickards, T. (1996). Assessing teacher-student interpersonal relationships in science classes. *Australian Science Teachers Journal*, 42, 28-33.
- Fosnot, C. T. (1996). *Constructivism: Theory, perspectives, and practice*. New York: Teachers College Press.

- Fraser, B.J. (1979). Evaluation of a science-based curriculum. In H.J. Walberg (Ed.), *Educational environments and effects: Evaluation, policy, and productivity* (pp. 218-234). Berkeley, CA: McCutchan.
- Fraser, B. J. (1981a). *Test of science-related attitudes*. Macquarie: Australian Council for Educational Research.
- Fraser, B. J. (1981b). Using environment assessments to make better classrooms. *Journal of Curriculum Studies*, 13, 131-144.
- Fraser, B. J. (1986). *Classroom environment*. London: Croom Helm.
- Fraser, B. J. (1990). *Individualized Classroom Environment Questionnaire*. Melbourne: Australian Council for Educational Research
- Fraser, B. J. (1994). Research on classroom and school climate. In D. Gabel (Ed.), *Handbook of research on science teaching and learning*, New York: Macmillan (pp. 493-541). New York: Macmillan.
- Fraser, B. J. (1998a). Science learning environments: Assessment, effects, and determinants. In B.J. Fraser & K. G. Tobins (Eds.), *International handbook of science education* (pp. 527-564). Dordrecht, The Netherlands: Kluwer.
- Fraser, B. J. (1998b). Classroom environment instruments: Development, validity, and applications. *Learning Environments Research*, 1, 7-33.
- Fraser, B. J. (2001). Twenty thousand hours: Editor's instruction. *Learning Environments Research*, 4, 1-5.
- Fraser, B. J. (2002). Classroom environments. In S.C. Goh & M.S. Khine (Eds.). *Studies in educational learning environments: An international perspective* (pp. 1-22). Singapore: World Scientific Publishing.
- Fraser, B. J., Anderson, G. J., & Walberg, H. J. (1982). *Assessment of learning environments: Manual for Learning Environment Inventory (LEI) and My Class Inventory (MCI)* (3<sup>rd</sup> vers.). Perth, Australia: Western Australian Institute of Technology.
- Fraser, B. J., & Chionh, Y. H. (2000, April). *Classroom environment, self-esteem, achievement, and attitudes in geography and mathematics in Singapore*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Fraser, B. J., & Fisher, D. L. (1986). Using short forms of classroom climate instruments to assess and improve classroom psychosocial environment. *Journal of Research in Science Teaching*, 5, 387-413.
- Fraser, B. J., Fisher, D. L., & McRobbie, C. J. (1996, April). *Development, validation, and use of personal and class forms of a new classroom environment instrument*. Paper presented at the annual meeting of the American Educational Research Association, New York.
- Fraser, B. J., Giddings, G. J., & McRobbie, C. J. (1995). Evolution and validation of a personal form of an instrument for assessing science laboratory classroom environments. *Journal of Research in Science Teaching*, 32, 399-422.
- Fraser, B. J., & Griffiths, A. K. (1992). Psychosocial environment of science laboratory classrooms in Canadian schools and universities. *Canadian Journal of Education*, 17, 391-404.
- Fraser, & McRobbie, C. J. (1995). Science laboratory classrooms at schools and universities: A cross-national study. *Educational Research and Evaluation: An International Journal on Theory and Practice*, 1, 1-25.
- Fraser, B. J., & O'Brien, P. (1985). Student and teacher perceptions of the environment of elementary-school classrooms. *Elementary School Journal*, 85, 567-580.

- Fraser, B. J., & Tobin, K. (1991). Combining qualitative and quantitative methods in classroom environment research. In B.J. Fraser & H.J. Walberg (Eds.), *Educational environments: Evaluation, antecedents and consequences* (pp. 271-291). Oxford, UK: Pergamon Press.
- Fraser, B. J., & Treagust, D. F. (1986). Validity and use of an instrument for assessing classroom psychosocial environment in higher education. *Higher Education*, 15, 37-57.
- Fraser, B. J., Treagust, D. F., & Dennis, N. C. (1986). Development of an instrument for assessing classroom psychosocial environment at universities and colleges. *Studies in Higher Education*, 11, 43-54.
- Fraser, B. J., & Walberg, H. J. (Eds.) (1991). *Educational environments: Evaluation, antecedents and consequences*. London: Pergamon.
- Fraser, B. J., & Wilkinson, W. J. (1993). Science laboratory classroom climate in British Schools and universities. *Research in Science & Technological Education*, 11, 49-70.
- French, A., & Chess, S. (2002). Canonical correlation and principal components analysis. Retrieved on March 23, 2005 from [userwww.sfsu.edu/~efu/classes/bio710/pca/ccandpca.htm](http://userwww.sfsu.edu/~efu/classes/bio710/pca/ccandpca.htm)
- Gallardo, A. (2002). The extension of the natural number domain to the integers in the transition from arithmetic to algebra. *Educational Studies in Mathematics*, 49, 171-192.
- Garin, E. (2003, April). *A study of schools participating in action research*. Paper presented at the annual meeting of the American Education Research Association, Chicago.
- Garin, E. (2005, April). *Teachers-as-researchers: Change or status quo?* Paper presented at the annual meeting of the American Education Research Association, Montreal, Canada.
- Glesne, C., & Peshkin, A. (1992). *Becoming qualitative researchers: An introduction*. White Plains, NY: Longman.
- Goh, S. C., & Fraser, B. J. (1995, April). *Learning environment and student outcome in primary mathematics classrooms in Singapore*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.
- Goh, S. C., & Khine, M. S. (Eds.) (2002). *Studies in educational learning environments: An international perspective*. Singapore: World Scientific Publishing.
- Goh, S., Young, D., & Fraser, B. (1995). Psychosocial climate and student outcomes in elementary mathematics classrooms: A multilevel analysis. *The Journal of Experimental Education*, 64, 29-40.
- Goos, M., Galbraith, P., & Renshaw, P. (1999). Establishing a community of practice in a secondary mathematics classroom. In L. Burton (Ed.), *Learning mathematics: From hierarchies to networks* (pp. 36-61). London: Falmer Press.
- Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003). Perspective on technology mediated learning in secondary school mathematics classrooms. *The Journal of Mathematical Behavior*, 22, 73-89.
- Greeno, J., Smith, D., & Moore, J. (1993). Transfer of situated learning. In D.K. Detterman, & R. J. Sternberg (Eds.), *Transfer on trial* (pp. 99-167). New York: Ablex Publishing.
- Guba, E. & Lincoln, Y. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage.

- Gullberg, J. (1997). *Mathematics from the birth of numbers*. New York: Norton.
- Harwell, S. H., Gunter, S., Montgomery, C. S., Shelton, C., & West, D. (2001). Technology integration and the classroom learning environment: Research for action. *Learning Environments Research*, 4, 259-286.
- Henderson, D., Fisher, D. L., & Fraser, B. J. (1995, April). *Gender differences in biology students' perceptions of actual and preferred learning environments*. Paper presented at the annual meeting of the National Association for Research in Science Teaching, San Francisco, CA.
- Huang, S. L., & Waxman, H. C. (1993, May). *Comparing Asian-and Anglo-American students' motivation and perceptions of the learning environment in mathematics*. Paper presented at the 15<sup>th</sup> annual conference of the National Association for Asian and Pacific American Education, New York.
- Hubbard, R., & Power, B. (1999). *Living the questions*. York, Maine: Stenhouse Publishers.
- Hughes-Hallett, D., Gleason, A., Flath, D., Gordon, S., Lomen, D., Lovelock, D., McCallum, W., Osgood, B., Pasquale, A., Techosky-Feldman, J., Thrash, K., & Tucker, T. (1994). *Calculus*. New York: Wiley.
- Huson, B., Lundin, D., & Samuels, N. (Eds.). (2003). *How to prepare for CAT/6: 8th grade*. Glendale, CA: Carney's Educational Services.
- Idiris, S., & Fraser, B. J. (1997). Psychosocial environment of agricultural science classrooms in Nigeria. *International Journal of Science Education*, 19, 79-91.
- Janvier, C. (1987). Conception and representations: The circle as an example. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 147-158). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Jegede, O., Fraser, B., & Fisher, D. (1995). The development and validation of a distance and open learning environment scale. *Educational Technology Research and Development*, 43, 90-93.
- Johnson, B., & McClure, R. (2004). Validity and reliability of a shortened, revised version of the constructivist learning environment survey (CLES). *Learning Environments Research*, 7, 65-80.
- Johnson, R. B., & Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher*, 33, 14-26.
- Kagan, D.M. (1992). Implications of research on teacher beliefs. *Educational Psychologist*, 27, 65-90.
- Kankkunen, M. (2001). Concept mapping and Peirce's semiotic paradigm meet in the classroom environment. *Learning Environments Research*, 4, 287-324.
- Kaput, J. J. (1995). Overcoming physicality and the external present: Cybernetic manipulatives. In R. Sutherland & J. Mason (Eds.), *Exploring mental imaging with computers in mathematics education* (pp. 161-177). Berlin: Springer Verlag.
- Kemmis, S., & McTaggart, R. (1988). *The action research planner*. Victoria, Australia: Deakin University Press.
- Kennedy, L., & Tipps, S. (2000). *Guiding children's learning of mathematics*. Belmont, CA: Wadsworth/Thompson Learning.
- Khoo, H. S., & Fraser B. J. (1997, April). *The learning environment associated with computer application courses for adults in Singapore*. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.
- Khoo, H. S., Fraser, B. J. (1998, April). *Using classroom environment dimensions in the evaluation of adult computer courses*. Paper presented at the annual

- meeting of the National Association for Research in Science Teaching, San Diego, CA.
- Kijkosol, D., & Fisher, D. L. (2005, August). *Gender, school situation and school size differences in students' perceptions of teacher-student interactions in biology classes in Thailand*. Paper presented at the Fourth International Conference on Science, Mathematics and Technology Education, Victoria, BC, Canada.
- Kim, H. B., Fisher, D. L., & Fraser, B. J. (1999). Assessment and investigation of constructivist science learning environments in Korea. *Research in Science and Technological Education*, 17, 239-249.
- Kim, H., Fisher, D., & Fraser, B. J. (2000). Classroom environment and teacher interpersonal behaviour in secondary science classes in Korea. *Evaluation and Research in Education*, 14, 3-22.
- Knobloch, E. (1994). Determinants. In I. Grattan-Guinness (Ed.), *Companion encyclopedia of the history and philosophy of the mathematical sciences* (pp. 766-209). London: Routledge.
- Koehler, M. S., & Grouws, D. A. (1992). Mathematics teaching practices and their effects. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 115-126). New York: Macmillan.
- Kordaki, M. (2003). The effect of tools of a computer microworld on students' strategies regarding the concept of conservation of area. *Educational Studies in Mathematics*, 52, 177-209.
- Kosinski, A. A. (2001). Cramer's rule is due to Cramer. *Mathematics Magazine*, 74, 310.
- Koul, R. B., & Fisher, D. L. (2005, August). *An investigation into teacher interaction, perceptions of learning environment, cultural differences and science achievement in India*. Paper presented at the Fourth International Conference on Science, Mathematics and Technology Education, Victoria, BC, Canada.
- Kuhs, T. (1994). Portfolio assessment: Making it work the first time. *Mathematics Teacher*, 87, 332-335.
- Lambdin, D. V., & Walker, V. L. (1994). Planning for classroom portfolio assessment. *Arithmetic Teacher*, 6, 318-324.
- Larson, R., Boswell, L., Kanold, T. D., & Stiff, L. (2001). *Algebra 1 concepts and skills*. Evanston, IL: McDougal Littell.
- Larson, R. E., Kanold, T. D., & Stiff, L. (1998). *Heath algebra 1*. Evanston, IL: McDougal Littell.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge University Press.
- Lee, S. S. U., & Fraser, B. J. (2001a, April). *High school science classroom learning environments in Korea*. Paper presented at the annual meeting of the National Association for Research in Science Teaching, St. Louis, MO.
- Lee, S. S. U., & Fraser, B. J. (2001b, December). *Science laboratory classroom environments in Korea*. Paper presented at the annual conference of the Australian Association for Research in Education, Fremantle, Australia.
- Lee, S. S. U., & Fraser, B. J. (2002, April). *High school science classroom learning environments in Korea*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Lewin, K. (1936). *Principles of topological psychology*. New York: McGraw.
- Little, J. W. (1984). Seductive images and organizational realities in professional



- development. *Teachers College Record*, 86, 84-102.
- Lorsbach, A., & Tobin, K. (1992). Research matters... To the science teacher: Constructivism as a referent for science teaching. *NARST Monograph*, 5, 21-27.
- Lucas, K., Roth, W. M. (1996). The nature of scientific knowledge and student learning: Two longitudinal case studies. *Research in Science Education*, 26, 103-127.
- MacAuley, D. J. (1990). Classroom environment: A literature review. *Educational Psychology*, 10, 239-253.
- Maher, C., & Alston, A. (1990). Teacher development in mathematics in a constructivist framework. In R. Davis, C. Maher, and N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 147-166). Reston, VA: National Council of Teachers of Mathematics.
- Majeed, A., Fraser, B. J., & Aldridge, J. M. (2002). Learning environment and its association with student satisfaction among mathematics students in Brunei Darussalam. *Learning Environments Research*, 5, 203-226.
- Mansfield, H. M., & Happs, J. C. (1992). Using grade eight students' existing knowledge to teach about parallel lines. *School Science and Mathematics*, 92, 450-454.
- Mansfield, H. M., & Happs, J. C. (1996). Using student conceptions of parallel lines to plan a teaching program. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics* (pp. 120-130). New York: Teacher College Press.
- Maor, D., & Fraser, B. J. (1996). Use of classroom perceptions in evaluating inquiry-based computer assisted learning. *International Journal of Science Education*, 18, 401-421.
- Margianti, E. S., Fraser, B. J., & Aldridge, J. M. (2001a, April). *Learning environment, mathematical ability and students' outcomes in university computing courses in Indonesia*. Paper presented at the annual meeting of the American Educational Research Association, Seattle, WA.
- Margianti, E. S., Fraser, B. J., & Aldridge, J. M. (2002, April). *Learning environment, attitudes and achievement: Assessing the perceptions of Indonesian university students*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Marjoribanks, K. (1991). Families, schools, and students' educational outcomes. In B. J. Fraser & H. J. Walberg (Eds.), *Educational environments: evaluation, antecedents and consequences* (pp. 75-91). London: Pergamon.
- Martin-Dunlop, C. (2003, January). *Improving understanding of the nature of science by integrating seabird research into a prospective elementary teachers' science course*. Paper presented at the annual meeting of Association for the Education of Teachers of Science, St. Louis, MO.
- Martin-Dunlop, C., & Fraser, B. J. (2005, August). *Improving the learning environment of university science courses: A key to better elementary teacher education*. Paper presented at the Fourth International Conference on Science, Mathematics and Technology Education, Victoria, BC, Canada.
- Masalski, W. J. (Ed.) (2005). *Technology-supported mathematics learning Environments: NCTM yearbook*. Reston, VA: National Council of Teachers of Mathematics.
- Matz, M. (1982). Towards a process model for high school algebra errors. In D.

- Sleeman & J. S. Browns (Eds.), *Intelligent tutoring systems* (pp. 25-50). New York: Academic Press.
- Maxwell, S. E., & Delaney, H. D. (1990). *Designing experiments and analyzing data: A model comparison perspective*. Belmont, CA: Wadsworth.
- McIntosh, M. (1991). No time for writing in your class? *Mathematics Teacher*, 84, 423-433.
- McRobbie, C. J., & Fraser, B. J. (1993). Associations between student outcomes and psychosocial science environment. *Journal of Educational Research*, 87, 78-85.
- Mink, D. V., Fraser, B. J. (2005). Evaluation of a K-5 mathematics program which integrates children's literature: Classroom environment and attitudes. *International Journal of Science and Mathematics Education*, 3, 59-85.
- Mittal, R. C., & Ahmad Al-Kurdi, A. (2001). Application of Cramer's rule in the solution of sparse systems of linear algebraic equations. *Journal of Computational and Applied Mathematics*, 136, 1-15.
- Moos, R. H. (1974). *The Social Climate Scales: An overview*. Palo Alto, CA: Consulting Psychologists Press.
- Moos, R. H. (1979). *Evaluating educational environments: Procedures, measures, findings, and policy implications*. San Francisco, CA: Jossey-Bass.
- Moos, R. H., & Trickett, E. J. (1987). *Classroom Environment Scale manual* (2nd ed.). Palo Alto, CA: Consulting Psychologists Press.
- Moschkovich, J. N. (1999). Students' use of the  $x$ -intercept as an instance of a transitional conception. *Educational Studies in Mathematics*, 37, 169-197.
- Murray, H. A. (1938). *Explorations in personality*. New York: Oxford University Press.
- National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.
- Newby, M., & Fisher, D. L. (1997). An instrument for assessing the learning environment of a computer laboratory. *Journal of Educational Computing Research*, 16, 179-190.
- Nix, R. K., Fraser, B. J., & Ledbetter, C. E. (2005). Evaluating an integrated science learning environment using the Constructivist Learning Environment Survey. *Learning Environments Research*, 8, 109-133.
- Nooriafshar, M. (2002). The use of innovative teaching methods for 'maximizing' the enjoyment from learning mathematics concepts. Faculty of Business, University of Southern Queensland, Australia. URL: <http://www.usq.edu.au/users/mehryar>.
- Norwood, K., Carter, G. (1994). Journal writing: An insight into students' understanding. *Teaching Children Mathematics*, 1, 146-48.
- Novak, J. D. (1990). Concept maps and Vee diagrams: Two metacognitive tools for science and mathematics education. *Instructional Science*, 19, 29-52.
- Novak, J. D. (1996). Concept mapping: A tool for improving science teaching and learning. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics* (pp. 32-34). New York: Teacher College Press.
- Novak, J. D., & Gowin, D. B. (1984). *Learning how to learn*. New York: Cambridge University Press.
- Novak J. D., & Musonda, D. (1991). A twelve-year longitudinal study of science concept learning. *American Educational Research Journal*, 28, 117-153.

- Novak, J. D., & Wandersee, J. (Guest Eds.). (1990). Concept mapping [Special issue]. *Journal of Research in Science Teaching*, 27, 923-1075.
- Olejnik, S., & Algina, J. (2000). Measures of effect size for comparative studies: Applications, interpretations, and limitations. *Contemporary Educational Psychology*, 25, 241-286.
- Onwuegbuzie, A. J., & Daniel, L. G. (2003). Typology of analytical and interpretational errors in quantitative and qualitative educational research. *Current Issues in Education*, 6(2). Retrieved June 29, 2005, from <http://cie.ed.asu.edu/volume6/number2>.
- Pace, C. R., & Stern, G. G. (1958). An approach to the measurement of psychological characteristics of college environments. *Journal of Educational Psychology*, 49, 269-277.
- Pauk, W. (1997). *How to study in college* (6th ed.). Boston: Houghton Mifflin Company.
- Peckham, P. D., Glass, G. V., & Hopkins, K. D. (1969). The experimental unit in statistical analysis: Comparative experiments with intact groups. *Journal of Special Education*, 3, 337-349.
- Peiro, M. M., & Fraser, B. J. (2005, August). *Science learning environments and student outcomes in early childhood grades*. Paper presented at the Fourth International Conference on Science, Mathematics and Technology Education, Victoria, BC, Canada.
- Pelton, L. F., & Pelton, T. W. (2005, August). *Supporting student participation in mathematics and science classrooms using an electronic constructed response system*. Paper presented at the Fourth International Conference on Science, Mathematics and Technology Education, Victoria, BC, Canada.
- Perneger, T.V. (1998). What is wrong with Bonferroni adjustments? *British Medical Journal*, 316, 1236-1238.
- Philippou, G. N., & Christou, C. (1998). The effects of a preparatory mathematics program in changing prospective teachers' attitudes towards mathematics. *Educational Studies in Mathematics*, 35, 189-206.
- Pirie, S. (1991). Peer discussion in the context of mathematical problem solving. In K. Durkin, & B. Shire (Eds.), *Language in mathematical education: Research and practice* (pp. 143-161). Philadelphia, PA: Open University Press.
- Post, T., Wachsmuth, I., Lesh, R., & Behr, M. (1985). Order and equivalence of rational numbers: A cognitive analysis. *Journal for Research in Mathematics Education*, 15, 18-36.
- Punch, K. F. (1998). *Introduction to social research*. London, England: Sage.
- Quek, C. L., Wong, A. F., & Fraser, B. J. (2005). Teacher-student interaction and gifted students' attitudes toward chemistry in laboratory classrooms in Singapore. *Journal of Classroom Interaction*, 40, 18-28.
- Raaflaub, C. A., & Fraser, B. J. (2002, April). *Investigating the learning environment in Canadian mathematics and science classrooms in which laptop computers are used*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Rentoul, A. J., & Fraser, B. J. (1979). Conceptualization of enquiry-based or open classroom learning environments. *Journal of Curriculum Studies*, 11, 233-245.
- Riah, H., & Fraser, B. J. (1998, April). *Chemistry learning environment and its*

- association with students' achievement in chemistry*. Paper presented at the annual meeting of the American Educational Research Association, San Diego, CA.
- Rickards, T., Bull, E., & Fisher, D. (2001). *Application of the WIHIC in a study of school racial diversity and socio-economic status*. Proceedings of the Western Australian Institute for Educational Research Forum 2000, Perth.
- Rickards, T., den Brok, P., Bull, E., & Fisher, D. (2003). *Predicting student views of the classroom: A Californian perspective*. Proceedings of the Western Australian Institute for Educational Research Forum 2003, Perth.
- Robinson, E., & Fraser, B. J. (2003, April). *Kindergarten students' and their parents' perceptions of science environments: Achievement and attitudes*. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.
- Robinson, W. S. (1950). Ecological correlations and the behavior of individuals. *American Sociological Society*, 15, 351-357.
- Ross, K. N. (1978). *Sample design for educational survey research* (Evaluation in Education Series). Oxford: Pergamon Press.
- Roth, W. M., & Bowen, G. (1995). Knowing and interacting: A study of culture, practices, and resources in a grade 8 open-ended science classroom guided by a cognitive apprenticeship model. *Cognition and Instruction*, 13, 73-128.
- Roth, W. M., & Roychoudhury, A. (1994). Physics students' epistemologies and views about knowing and learning. *Journal of Research in Science Teaching*, 31, 5-30.
- Saljo, R. (1979). *Learning in the learner's perspective: Some commonsense conceptions*. Gothenburg, Sweden: Institute of Education, University of Gothenburg.
- Sanchez, J. C., Encinas, L. H., Fernandez, R. L., Sanchez, M. R., (2002). Designing hypermedia tools for solving problems in mathematics. *Computers & Education*, 38, 303-317.
- Santiboon, T. (2005, August). *Learning environments and teacher-student interactions in physics classes in Thailand*. Paper presented at the Fourth International Conference on Science, Mathematics and Technology Education, Victoria, BC, Canada.
- Saxon, J. H. (Ed.) (1998). *Algebra 1: An incremental development*. Norman, OK: Saxon Publishers.
- Sebela, M. P., Fraser, B. J., & Aldridge, J. M. (2004). Using teacher action research to promote constructivist learning environments in South Africa. *South African Journal of Education*, 24, 245-253.
- Sharan, R. E. (1980). Cooperative learning. *Review of Educational Research*, 50, 315-342.
- Simonsen, L. M., & Dick, T. P. (1997). Teachers' perceptions of the impact of graphing calculators in the mathematics classroom. *Journal of Computers in Mathematics and Science Teaching*, 16, 239-268.
- Sinclair, B. B., & Fraser, B. J. (2002). Changing classroom environments in urban middle schools. *Learning Environments Research: An International Journal*, 5, 301-328.
- Slavin, R. E. (1987). *Cooperative learning: Student teams* (2nd ed.). Washington, DC: National Educational Association.
- Slavit, D. (1999). The role of operation sense in transitions from arithmetic to algebraic thought. *Educational Studies in Mathematics*, 37, 251-274.

- Soerjaningsih, W., Fraser, B. J. & Aldridge, J. M. (2001a, April). *Achievement, satisfaction and learning environment among Indonesian computing students at the university level*. Paper presented at the annual meeting of the American Educational Research Association, Seattle, WA.
- Solomon, J. (1985). Classroom discussion: A method of research for teachers? *British Educational Research Journal*, 11, 153-162.
- Spinner, H., & Fraser, B. J. (2005). Evaluation of an innovative mathematics program in terms of classroom environment, students' attitudes and conceptualization. *International Journal of Science and Mathematics Education*, 3, 267-293.
- Steffe, L. P., & D'Ambrosio, B. S. (1996). Understanding teaching experiment to enhance understanding of students' mathematics. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.), *Improving teaching and learning in science and mathematics* (pp. 66-7). New York: Teacher College Press.
- Stein, G. G., Stein, M. I., & Bloom, B. S. (1956). *Methods in personality assessment*. Glencoe, IL: Free Press.
- Stenhouse, L. (1975). *An introduction to curriculum research and development*. London: Heinemann.
- Stenmark, J. K. (1989). *Assessment alternatives in mathematics: An overview of assessment techniques that promote learning*. Berkeley, CA: Lawrence Hall of Science.
- Stipek, D., Salmon, J. M., Givvin, K. B., Kazemi, E., Saxe, G., & Macgyvers, V. L. (1998). The value (and convergence) of practices suggested by motivation research and promoted by mathematics education reformers. *Journal for Research in Mathematics Education*, 29, 465-488.
- Sweller, J. Mawer, R. F., & Ward, M. R. (1983). Development of expertise in mathematical problem solving. *Journal of Experimental Psychology: General*, 112, 639-61.
- Taylor, P. C. Dawson, V., & Fraser, B. J. (1995, April). *Classroom learning environments under transformation: A constructivist perspective*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.
- Taylor, P. C., Fraser, B. J., & Fisher, D. L. (1997). Monitoring constructivist classroom learning environments. *International Journal of Educational Research*, 27, 293-302.
- Taylor, P. C., Fraser, B. J., & White, L. R. (1994, April). *The revised CLES: A questionnaire for educators interested in the constructivist reform of school science and mathematics*. Paper presented at the annual meeting of the American Educational Research Association, Atlanta, GA.
- Teh, G., & Fraser, B. J. (1994). An evaluation of computer-assisted learning in terms of achievement, attitudes and classroom environment. *Evaluation and Research in Education*, 8, 147-161.
- Teh, G., & Fraser, B. J. (1995). Development and validation of an instrument for assessing the psychosocial environment of computer-assisted learning classrooms. *Journal of Educational Computing Research*, 12, 177-193.
- Tharp, M. L., Fitzsimmons, J. A., & Ayers, R. L. B. (1997). Negotiating a technological shift: Teacher perception of the implementation of graphing calculators. *Journal of Computers in Mathematics and Science Teaching*, 16, 551-575.

- Thomas, M., Tyrrell, J., & Bullock, J. (1996). Using computers in the mathematics classroom: The role of the teacher. *Mathematics Education Research Journal*, 7, 38-57.
- Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.
- Thompson, B. (1998a). Review of 'what if there were no significance tests?' *Educational and Psychological Measurement*, 58, 334-346.
- Thompson, B. (1998b, April). *Five methodology errors in educational research: The pantheon of statistical significance and other faux pas*. Invited address presented at the annual meeting of the American Educational Research Association, San Diego, CA.
- Thorp, H., Burden, R. L., & Fraser, B. J. (1994). Assessing and improving classroom environment. *School Science Review*, 75, 107-113.
- Tobin, K., & Fraser, B. J. (1998). Qualitative and quantitative landscapes of classroom learning environments. In B. J. Fraser & K. C. Tobin (Eds.), *The international handbook of science education* (pp. 623-640). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Tobin, K., Kahle, J. B., & Fraser, B. J. (Eds.). (1990). *Windows into science classrooms: Problems associated with higher-level cognitive learning*. London: Falmer Press.
- Treagust, D. F., Duit, R., & Fraser, B. J. (Eds.). (1996). *Improving teaching and learning in science and mathematics*. New York: Teacher College Press.
- Treagust, D. F., Jacobowitz, R., Gallagher, J. J., & Parker, J. (2003). Embed assessment in your teaching. *Science Scope*, 26, 36-39.
- Trickett, E. J., & Moos, R. H. (1973). Social environment of junior high and high school classrooms. *Journal of Educational Psychology*, 65, 93-102.
- Trigwell, K., & Prosser, M. (1991). Relating approaches to study and quality of learning outcomes at the course level. *British Journal of Educational Psychology*, 61, 265-275.
- United States Department of Education. (n.d.). No Child Left Behind Act. Retrieved February 5, 2005, from <http://www.ed.gov/nclb/methods/math/math.pdf>.
- Urban-Lurain, M. (2001, April). *Teaching FITness for conceptual understanding: A computer science course for non-computer majors*. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.
- Van Rossum, E., & Schenk, S. (1984). The relationship between learning conception, study strategy and learning outcome. *British Journal of Educational Psychology*, 54, 73-83.
- Veenman, S., Denessen, D., van den Akker, A., & van der Rijt, J. (2005). Effects of a cooperative learning program on the elaborations of students during help seeking and help giving. *American Educational Research Journal*, 42, 115-151.
- Von Saldern, M. (1992). *Social climate in the classroom: Theoretical and methodological aspects*. New York: Waxmann Munster.
- Waggett, D. (2001, January). *Secondary science teacher candidates' beliefs and practices*. Paper presented at the international meeting of the Association for the Education of Teachers in Science, Costa Mesa, CA.
- Walberg, H. J. (Ed.). (1979). *Educational environments and effects: Evaluation, policy, and productivity*. Berkeley, CA: McCutchan.

- Walberg, H. J., & Anderson, G. J. (1968). Classroom climate and individual learning. *Journal of Educational Psychology*, 59, 414-419.
- Waldrip, B. G., & Fisher, D. L. (2000, April). *City and country students' perceptions of teacher-student interpersonal behavior and classroom learning environments*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Watters, J., & Ginns, I. (1994). Self-efficacy and science anxiety among preservice primary teachers: Origins and remedies. *Research in Science Education*, 24, 348-357.
- Watzlawick, P., Beavin, J., & Jackson, D. (1967). *The pragmatics of human communication*. New York: Norton.
- Wildt, A. R., & Olli, T. A. (1978). *Analysis of covariance* (Quantitative applications in the social sciences series #12). Thousand Oaks, CA: Sage Publications.
- Wilkinson, L., Hill, M., & Vang, E. (1992). *SYSTAT: Statistics, Version 5.2*. Evanston, IL: SYSTAT, Inc.
- Wilks, D. R. (2000). *An evaluation of classroom learning environments using critical constructivist perspectives as a referent for reform*. Unpublished doctoral thesis, Curtin University of Technology, Perth, Australia.
- Wong, A. L. F., & Fraser, B. J. (1994, April). *Science laboratory classroom environments and student attitudes in chemistry classes in Singapore*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.
- Wong, A. L. F., & Fraser, B. J. (1996). Environment-attitude associations in the chemistry laboratory classroom. *Research in Science and Technological Education*, 14, 91-102.
- Wong, A. F. L., Young, D. J., & Fraser, B. J. (1997). A multilevel analysis of learning environments and student attitudes. *Educational Psychology*, 17, 449-468.
- Wong, N. Y. (1993). Psychosocial environments in the Hong Kong mathematics classroom. *Journal of Mathematical Behavior*, 12, 303-309.
- Wong, N.Y. (1996). Students' perceptions of the mathematics classroom in Hong Kong. *Hiroshima Journal of Mathematics Education*, 4, 89-107.
- Wubbels, T. (1993). *Teacher-student relationships in science and mathematics classes* (What research says to the science and mathematics teacher, No 11). Perth: National Key Center for School Science and Mathematics, Curtin University of Technology.
- Wubbels, T., Creton, H. A., & Hooymayers, H. P. (1992). Review of research on Teacher communication styles with use of the Leary model. *Journal of Classroom Interaction*, 27, 1-12.
- Wubbels, T., & Levy, J. (1993). *Do you know what you look like?: Interpersonal relationships in education*. London: Falmer Press.
- Yarrow, A. Millwater, J., & Fraser, B. J. (1997). Improving university and primary school classroom environments through preservice teachers' action research. *International Journal of Practical Experiences in Professional Education*, 1, 68-93.
- Zandvliet, D. B., & Fraser, B. J. (2004). Learning environments in information and communication technology classrooms. *Technology, Pedagogy and Education*, 13, 97-124.
- Zandvliet, D. B., & Fraser, B. J. (2005). Physical and psychosocial environments

associated with networked classrooms. *Learning Environments Research*, 8, 1-17.





## **Appendix A**

### **Constructivist Learning Environment Survey**

The questionnaire in Appendix A consists of three scales selected from the Constructivist Learning Environment Survey (CLES) developed by Taylor, Fraser and Fisher (1997). See Section 3.4.1 for more details. The CLES was used in my study and is included in this appendix with permission of the authors.

## Constructivist Learning Environment Survey

### Directions for Students

These questionnaires contain statements about practices which could take place in this class. You will be asked how often each practice takes place.

There are no 'right' or 'wrong' answers. Your opinion is what is wanted. Think about how well each statement describes what this class is like for you.

Draw a circle around

- |          |                             |                      |
|----------|-----------------------------|----------------------|
| <b>1</b> | if the practice takes place | <b>Almost Never</b>  |
| <b>2</b> | if the practice takes place | <b>Seldom</b>        |
| <b>3</b> | if the practice takes place | <b>Sometimes</b>     |
| <b>4</b> | if the practice takes place | <b>Often</b>         |
| <b>5</b> | if the practice takes place | <b>Almost Always</b> |

Be sure to give an answer for all questions. If you change your mind about an answer, just cross it out and circle another.

Some statements in this questionnaire are fairly similar to other statements. Don't worry about this. Simply give your opinion about all statements.

### Practice Example

Suppose you were given the statement 'I choose my partners for group discussion'. You would need to decide whether you choose your partners 'Almost always', 'Often', 'Sometimes', 'Seldom' or 'Almost Never'. If you selected 'Often', then you would circle the number 2 on your questionnaire.

<b><i>Learning about the world</i></b>	Almost Never	Seldom	Some- times	Often	Almost Always
In this class . . .					
1. I learn about the world outside of school.	1	2	3	4	5
2. My new learning starts with problems about the world outside of school.	1	2	3	4	5
3. I learn how math can be part of my out-of-school life.	1	2	3	4	5
In this class . . .					
4. I get a better understanding of the world outside of school.	1	2	3	4	5
5. I learn interesting things about the world outside of school.	1	2	3	4	5
6. What I learn has nothing to do with my out-of-school life.	1	2	3	4	5
<b><i>Learning to learn</i></b>	Almost Never	Seldom	Some- times	Often	Almost Always
In this class . . .					
7. I help the teacher to plan what I'm going to learn.	1	2	3	4	5
8. I help the teacher to decide how well I am learning.	1	2	3	4	5
9. I help the teacher to decide which activities are best for me.	1	2	3	4	5
In this class . . .					
10. I help the teacher to decide how much time I spend on learning activities.	1	2	3	4	5
11. I help the teacher to decide which activities I do.	1	2	3	4	5
12. I help the teacher to assess my learning.	1	2	3	4	5
<b><i>Learning to communicate</i></b>	Almost Never	Seldom	Some- times	Often	Almost Always
In this class . . .					
13. I get the chance to talk to other students.	1	2	3	4	5
14. I talk with other students about how to solve problems.	1	2	3	4	5
15. I explain my understandings to other students.	1	2	3	4	5
In this class . . .					
16. I ask other students to explain their thoughts.	1	2	3	4	5
17. Other students ask me to explain my ideas.	1	2	3	4	5
18. Other students explain their ideas to me.	1	2	3	4	5

## **Appendix B**

### **What Is Happening In this Class? (WIHIC) Questionnaire**

The questionnaire in Appendix B consists of three scales selected from the What Is Happening In this Class? (WIHIC) developed by Fraser, Fisher and McRobbie (1996). See Section 3.4.2 for further information. The WIHIC was used in my study and is included in this appendix with the permission of the authors.

### What Is Happening In this Class? (WIHIC) Questionnaire

#### INSTRUCTIONS:

Use a blue/black pen or a 2B pencil.

#### SECTION A. Background Information

	Yes	No
Does your family have a computer at home?	<input type="radio"/>	<input type="radio"/>
Do you use our home computer for school-related work?	<input type="radio"/>	<input type="radio"/>
Do you have access to the Internet at home?	<input type="radio"/>	<input type="radio"/>
Would your parent(s) like you to go to university after you leave school?	<input type="radio"/>	<input type="radio"/>
Do you intend to go to university after you leave school?	<input type="radio"/>	<input type="radio"/>

#### What type of job would you like when you leave school?

(Fill in ONE circle only or, if your preferred job is not listed, select "Other" and provide details.)

<input type="radio"/> Vet	<input type="radio"/> Teacher	<input type="radio"/> Accountant	<input type="radio"/> Shop-keeper	<input type="radio"/> Other (please specify)
<input type="radio"/> Doctor	<input type="radio"/> Journalist	<input type="radio"/> Nurse	<input type="radio"/> Dentist	
<input type="radio"/> Lawyer	<input type="radio"/> Builder	<input type="radio"/> Chef	<input type="radio"/> Model	
<input type="radio"/> Scientist	<input type="radio"/> Pilot	<input type="radio"/> Sports person	<input type="radio"/> Fashion Designer	
<input type="radio"/> Programmer	<input type="radio"/> Flight Attendant	<input type="radio"/> Physiotherapist	<input type="radio"/> Banker	
<input type="radio"/> Actor	<input type="radio"/> Pharmacist	<input type="radio"/> Psychologist	<input type="radio"/> Don't know	

Subject:  Grade: ☐ 6 ☐ 8 ☐ 10 ☐ 12  
☐ 7 ☐ 9 ☐ 11

School:  Sex: ☐ Male ☐ Female

#### SECTION B.

This section contains statements about practices that could take place in this class. You will be asked how often each practice takes place. There are no 'right' or 'wrong' answers. Your opinion is what is wanted. Your responses will be confidential.

The 'Actual' column is to be used to describe how often each practice actually takes place in your class. The 'Preferred' column is to be used to describe how often you would like each practice to take place (a wish list).

IN	ACTUAL					PREFERRED				
	Almost	Seldom	Some-	Often	Almost	Almost	Seldom	Some-	Often	Almost

	Never	↑	→	↓	Always	Never	↑	→	↓	Always
17. I discuss ideas in class.	←	↑	→	↓	○	←	↑	→	↓	○
18. I give my opinions during class discussions.	←	↑	→	↓	○	←	↑	→	↓	○
19. The teacher asks me questions.	←	↑	→	↓	○	←	↑	→	↓	○
20. My ideas and suggestions are used during classroom discussions.	←	↑	→	↓	○	←	↑	→	↓	○
21. I ask the teacher questions.	←	↑	→	↓	○	←	↑	→	↓	○
22. I explain my ideas to other students.	←	↑	→	↓	○	←	↑	→	↓	○
23. Students discuss with me how to go about solving problems.	←	↑	→	↓	○	←	↑	→	↓	○
24. I am asked to explain how I solve problems.	←	↑	→	↓	○	←	↑	→	↓	○

TO	ACTUAL					PREFERRED				
	Almost Never	Seldom	Some- Times	Often	Almost Always	Almost Never	Seldom	Some- Times	Often	Almost Always
25. Getting a certain amount of work done is important to me.	←	↑	→	↓	○	←	↑	→	↓	○
26. I do as much as I set out to do.	←	↑	→	↓	○	←	↑	→	↓	○
27. I know the goals for this class.	←	↑	→	↓	○	←	↑	→	↓	○
28. I am ready to start this class on time.	←	↑	→	↓	○	←	↑	→	↓	○
29. I know what I am trying to accomplish in this class.	←	↑	→	↓	○	←	↑	→	↓	○
30. I pay attention during this class.	←	↑	→	↓	○	←	↑	→	↓	○
31. I try to understand the work in this class.	←	↑	→	↓	○	←	↑	→	↓	○
32. I know how much work I have to do.	←	↑	→	↓	○	←	↑	→	↓	○

SECTION B. Continued...

IV	ACTUAL					PREFERRED				
	Almost Never	Seldom	Some- Times	Often	Almost Always	Almost Never	Seldom	Some- Times	Often	Almost Always
33. I carry out investigations	←	↑	→	↓	○	←	↑	→	↓	○

to test my ideas.	←	↑	→	↓	○	←	↑	→	↓	○
34. I am asked to think about the evidence for statements.	←	↑	→	↓	○	←	↑	→	↓	○
35. I carry out investigations to answer questions coming from discussions.	←	↑	→	↓	○	←	↑	→	↓	○
36. I explain the meaning of statements, diagrams, and graphs.	←	↑	→	↓	○	←	↑	→	↓	○
37. I carry out investigations to answer questions that puzzle me.	←	↑	→	↓	○	←	↑	→	↓	○
38. I carry out investigations to answer the teacher's questions.	←	↑	→	↓	○	←	↑	→	↓	○
39. I find out answers to questions by doing investigations.	←	↑	→	↓	○	←	↑	→	↓	○
40. I solve problems by using information obtained from my own investigations.	←	↑	→	↓	○	←	↑	→	↓	○

*Thank you for your assistance in completing this questionnaire.*



## **Appendix C**

### **Test of Mathematics-Related Attitudes**

The questionnaire is based on two scales from the Test of Science-Related Attitudes (TOSRA) developed by Fraser (1981a). See Section 3.4.3 for further details. TOSRA was adapted and used in my study and is included in the appendix with the permission of the author.

# TOMRA

## TEST OF MATHEMATICS-RELATED ATTITUDES

### DIRECTIONS

1. This test contains a number of statements about math. You will be asked what you yourself think about these statements. There are no ‘right’ or ‘wrong’ answers. Your opinion is what is wanted.

2. For each statement, draw a circle around

SA if you **STRONGLY AGREE** with the statement;

A if you **AGREE** with the statement;

N if you are **NOT SURE**;

D if you **DISAGREE** with the statement;

SD if you **STRONGLY DISAGREE** with the statement.

### Practice Item

0 It would be interesting to learn about boats.

Suppose that you **AGREE** with this statement, then you would circle A on your Answer Sheet, like this:

0 SA (A) N D SD

4. If you change your mind about an answer, cross it out and circle another one.

5. Although some statements in this test are fairly similar to other statements, you are asked to indicate your opinion about all statements.

1. Mathematicians usually like work on math problems when they have a day off.	SA	A	N	D	SD
2. Math lessons are fun.	SA	A	N	D	SD
3. Mathematicians are about as fit and healthy as other people.	SA	A	N	D	SD
4. I dislike math lessons.	SA	A	N	D	SD
5. Mathematicians do not have enough time to spend with their families.	SA	A	N	D	SD
6. School should have more math lessons each week.	SA	A	N	D	SD
7. Mathematicians like sport as much as other people do.	SA	A	N	D	SD
8. Math lessons bore me.	SA	A	N	D	SD
9. Mathematicians are less friendly than other people.	SA	A	N	D	SD
10. Math is one of the most important school subjects.	SA	A	N	D	SD
11. Mathematicians can have a normal family life.	SA	A	N	D	SD
12. Math lessons are a waste of time.	SA	A	N	D	SD
13. Mathematicians do not care about their working conditions.	SA	A	N	D	SD
14. I really enjoy going to math lessons.	SA	A	N	D	SD
15. Mathematicians are just as interested in art and music as other people are.	SA	A	N	D	SD
16. The material covered in math lessons is uninteresting.	SA	A	N	D	SD
17. Few mathematicians are happily married.	SA	A	N	D	SD
18. I look forward to math lessons.	SA	A	N	D	SD
19. If you met a mathematician, he would probably look like anyone else you might meet.	SA	A	N	D	SD
20. I would enjoy school more if there were no math lessons.	SA	A	N	D	SD

## **Appendix D**

### **Post-test on Systems of Linear Equations**

Some of the questions were adapted from Larson, Boswellll, Kanold and Stiff (2001).

Post-test on Systems of Linear Equations

Name:\_\_\_\_\_ Per:\_\_\_\_\_ Date:\_\_\_\_\_

1. Explain what it means for the ordered pair  $(a,b)$  to be a solution to the linear equation  $Ax + By = C$
  
  
  
  
  
  
  
  
  
  
2. How is a solution of a linear system similar to a solution of a linear equation?  
How is it different?
  
  
  
  
  
  
  
  
  
  
3. An ordered pair  $(a,b)$  may be a solution to a linear equation but not a solution of the linear system. Explain.
  
  
  
  
  
  
  
  
  
  
4. Is the ordered pair a solution of the equation  $3x - 2y = 11$   $(5,2)$ ,  $(-1, 1)$ ,  $(1,-1)$
  
  
  
  
  
  
  
  
  
  
5. Explain what it means to solve a linear system using the graph –and –check method.

## **Appendix E**

### **Multiple-Choice Questions on Systems of Linear Equations**

Some items were adapted from Huson, Lundin, and Samuels (2003, pp. 91-92).

**Multiple Choice Questions**

Solve questions 1-4 algebraically.

1.  $3x - y = 9$  and  $x - y = 5$

- a.  $\left(\frac{7}{2}, \frac{-3}{2}\right)$
- b.  $(7, 2)$
- c.  $(1, -6)$
- d.  $(2, -3)$

3.  $4x + 3y = 5$  and  $2x - 5y = 9$

- a.  $(-2, 1)$
- b.  $(-2, -1)$
- c.  $(1, -6)$
- d.  $(5, 5)$

2.  $x = 2y - 1$  and  $2x - 3y = -4$

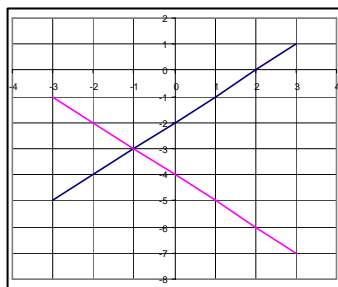
- a.  $(-3, -7)$
- b.  $(-5, -2)$
- c.  $(3, 5)$
- d.  $(2, 3)$

4.  $3x + 2y = 10$  and  $y = -4x + 15$

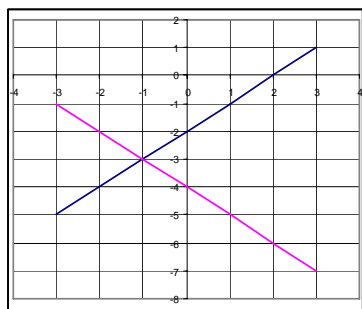
- a.  $(2, -2)$
- b.  $(4, -1)$
- c.  $(3, 5)$
- d.  $(2, 3)$

Solve questions 5 and 6 by graphing.

5.  $y = -x + 2$  and  $3x - y = -2$



6.  $x - y = -3$  and  $y = -x - 1$



7. Where do the graphs of  $y = x + 1$  and  $y = 2x - 5$  intersect?

- a. (4, 3)
- b. (3, 4)
- c. (7, 6)
- d. (6, 7)

Solve the system of equations by any method.

8.  $y = -2x$   
 $y = x + 3$

- a. (-1, 2)
- b. (-3, 0)
- c. (2, -1)
- d. (0, -3)

9.  $-3x + y = -1$   
 $-2x + y = 11$



- a.  $(-5, 0)$
- b.  $(0, -5)$
- c.  $(5, 2)$
- d.  $(2, 5)$

10.  $x = 4$   
 $y = -2$

- a.  $(-2, 2)$
- b.  $(2, -2)$
- c.  $(4, -2)$
- d.  $(-2, 4)$

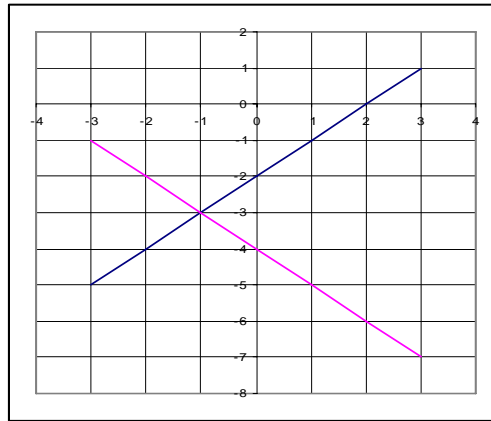
## **Appendix F**

### **Second Version of Multiple-Choice Test Questions on Systems of Linear Equations**

Test items adapted from Larson, Boswell, Kanold and Stiff (2001, p. 436).

### Multiple Choice Test Questions

1. Which point appears to be the solution of the linear system graphed below?



- (A)  $(-4, 0)$   
 (B)  $(-3, -1)$   
 (C)  $(-1, -3)$   
 (D)  $(0, -2)$
2. The ordered pair  $(3, 4)$  is a solution of which linear system?

- (A)  $x + y = 7$   
 $x + 2y = 11$   
 (C)  $x - y = 1$   
 $2x + y = 10$   
 (B)  $x - y = 1$   
 $2x - y = 9$   
 (D)  $x + y = 7$   
 $2x - 2y = 14$

3. What is the solution of the following linear system?

$$\begin{aligned} -2x + 7y &= -3 \\ x - 7y &= -2 \end{aligned}$$

- (A) 1  
 (B) 5  
 (C)  $(1, 5)$   
 (D)  $(5, 1)$

4. What is the solution of the following linear system?

$$\begin{aligned} 5x - 6y &= -10 \\ -15x + 14y &= 10 \end{aligned}$$

- (A)  $(-5, -8)$  (B)  $(-2, 0)$   
 (C)  $(4, 5)$  (D)  $(10, 10)$

5. You have 50 ride tickets. You need 3 tickets to ride the Ferris wheel and 5 tickets to ride the roller coaster. You ride 12 times. How many times did you ride the roller coaster?

(A) 5 (B) 7  
(C) 10 (D) 18

6. How many solutions does the following linear system have?

$$4x - 2y = 6$$

$$2x - y = 3$$

(A) One  
(B) Two  
(C) Infinitely many  
(D) None

7. Which system of linear equations has no solution?

(A)  $y = 2x + 4$   
 $y = 2$   
 $3x + 4y = 10$   
(B)  $3x + 2y = 8$   
 $5x + 2y = 11$   
(C)  $10x + 4y = 11$   
 $2x - 4y = -5$   
(D)  $3x + 6y = 15$   
(E) None of these

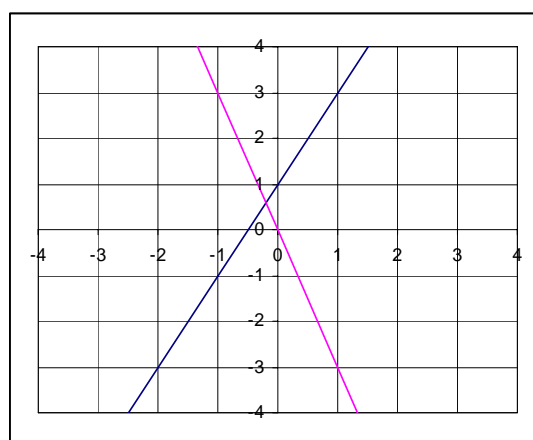
8. Which point is a solution of the following system of linear inequalities?

$$y < -x$$

$$y < x$$

(A) (6, -2) (B) (-2, 6)  
(C) (-1, -6) (D) (-6, -1)

9. Which system of inequalities is represented by the graph below?



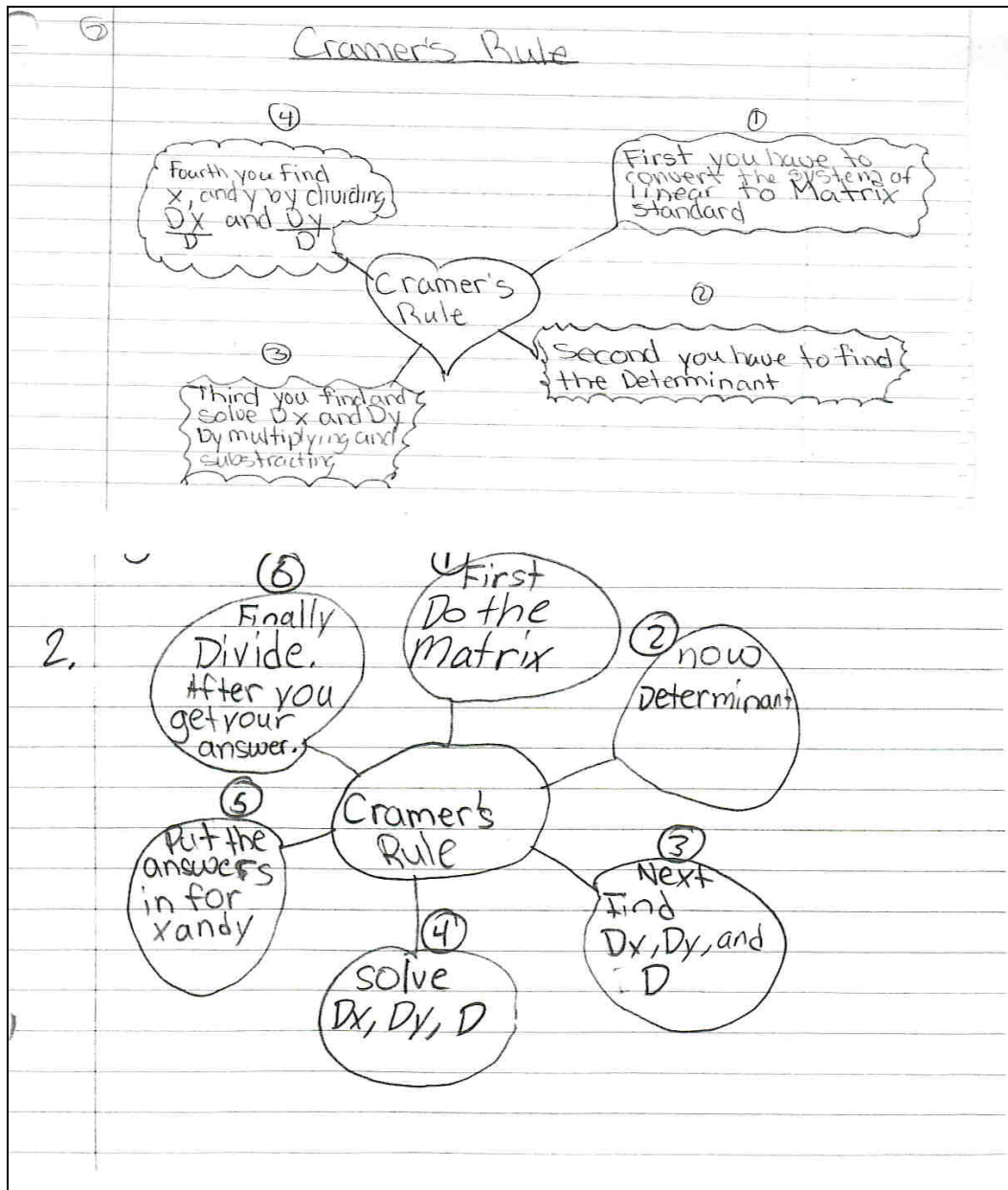
(A)  $y \leq 2x + 1$   
 $2y < x$   
(B)  $y \geq 2x + 1$   
 $2y > -3x$   
(C)  $y \leq 2x + 1$   
 $2y < 3x$   
(D)  $y \geq 2x + 1$   
 $2y > 3x$

These questions are adapted from McDougal Littell, Concepts and Skills, California Teacher's Edition (2001, p. 436).

## **Appendix G**

### **Concept Maps from Students' Journals**

# Concept Maps



The concept map samples obtained from my study as part of students' journal